Adaptation of Morphological Masks by Genetic Algorithms

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Abstract

Sequences of basic operations of mathematical morphology can perform high-level image processing tasks. Morphological operations make use of a so-called structuring element, which should be adapted to improve the performance of certain tasks. A serious problem arises from the fact that greyscale morphology influences the grey-value distribution in the image in an unpredictable manner. This problem was not addressed in literature so far. This paper introduces a new fitness function that overcomes this disadvantage and offers new possibilities for the adaptation of structuring elements in mathematical morphology. The new approach is based on the comparison of performing the same operations sequence on the starting image and a scattered variant of this image. The more this two images differ the better the fitness of the structuring element. Also a method is introduced to estimate the histogram of the operated scattered image. A result is given for the application of this method in texture analysis.

Introduction

Morphological operations based on formal combinations of the basic morphological operations erosion and dilation gives new ways for the solution of image processing tasks. Several kinds of basic erosion and dilation definitions are available, e.g. binary morphology, grey-scale morphology and three-dimensional morphology [1].

Dilation and erosion are defined by means of set operations and a structuring element M. The structuring element is a set of offsets. Suppose there is a binary image described by the set of foreground pixels (i.e. I = {x| x is foreground} and I , the image function, is a subset of NxN), then erosion of I by the structuring element M is defined by (´+´ stands for addition of indices):

(1) $$\varepsilon_M(I) = \{x \mid \underset{y \in M}{\forall} \{(x+y) \in I\}\}$$

i.e. the set of foreground pixels is eroded by the shape of the structuring element M. Dilation is defined by the inverted structuring element $\overline{M} = \{x \mid -x \in M\}$ as follows:

(2) $$\delta_M(I) = \{x \mid \underset{y \in \overline{M}}{\exists} (x+y) \in I\}\}$$

i.e. the shape of the foreground is dilated by the shape of the structuring element M.

A proper way of establishing complex operations is by means of dilation and erosion in operation sequences like

a b

figure 1: Textile example (a) and eroded image (b)

(3) $\rho_M(I) = \delta_M(I) - \varepsilon_M(I)$

for the definition of the inner gradient _ of the image function I as grey-value difference of the dilation δ and the erosion ε with a certain structuring element M at every image position x.
There has been several attempts to use the formal simplicity of morphological operations to perform high-level image processing tasks by the adaptation of a sequence of certain morphological operations [2]. It is not a new idea to adapt the index set of a structuring element M (also called morphological mask) for a certain operation sequence by means of genetic algorithm [3,4]. This is a consequence of the straightforward encoding of a morphological mask into a genom. This task can be easy performed by embedding the mask as set of indices into a superset of indices with a regular shape (like a 3x3-mask). In the following a genom of the size of mask positions of the superset can be used and for every mask position set in the mask the corresponding bit in the genom is set to 1. For a 3x3-mask the corresponding genom has size 9 bit.

Adaptation of masks without using genetic algorithms can be found in literature [2]. They are mainly based on extrema calculation of the image function.

But a main problem of adapting masks to image functions has not been addressed so far. Taking definitions (1) and (2) for erosion and dilation this definitions have to be expanded to grey-scaled image functions. Erosion and dilation for a grey-scaled image functions $I(x)$, formally defined as:

(4) $\varepsilon_M(I)(x) = \min_{y \in M}(I(x+y))$

for erosion and:

(5) $\delta_M(I)(x) = \max_{y \in M}(I(x+y))$

for dilation are strongly modifying the grey-value distribution of the original image if they have more than two grey-values (i.e. are not pseudo-binary images). Consider figure 1, where a textile example is shown.
The textile is irregularly to a low degree, so the problem arises of finding a mask which best matches the overall regularity of the texture. Because this is an optimization task, a teachable method like genetic algorithm can be just to adapt a morphological mask due to the optimization criterion (6):

$$V_M(G) = \sum_{x \in G} (G(x) - \varepsilon_{M;G}(x)) \rightarrow MIN$$

Consider figure 2 where the histogram before and after erosion of the image of figure 1 can be seen.
The distribution of grey-values is shifted to the part of lower grey-values. This shift depends on several factors like the shape of the mask, the grey-value distribution of the original image and the local grey-value neighborhoods contained in the image. This gives a problem for the fitness function because different choices for the mask positions can´t be compared due to its unpredictable influence on the image histogram.
This problem was not addressed in recent publications on that subject, because only binary morphology was used [3] or because the optimization goal did not match clear defined fitness measurements (like "noise reduction") [2,4]. For this approaches it was sufficient to take as fitness function the volume difference to the original image after morphological processing.
This paper introduces a new approach to overcome this problem by defining a better fitness function which could be generally expanded to every case of adaptation of morphological sequences to certain image filtering tasks.

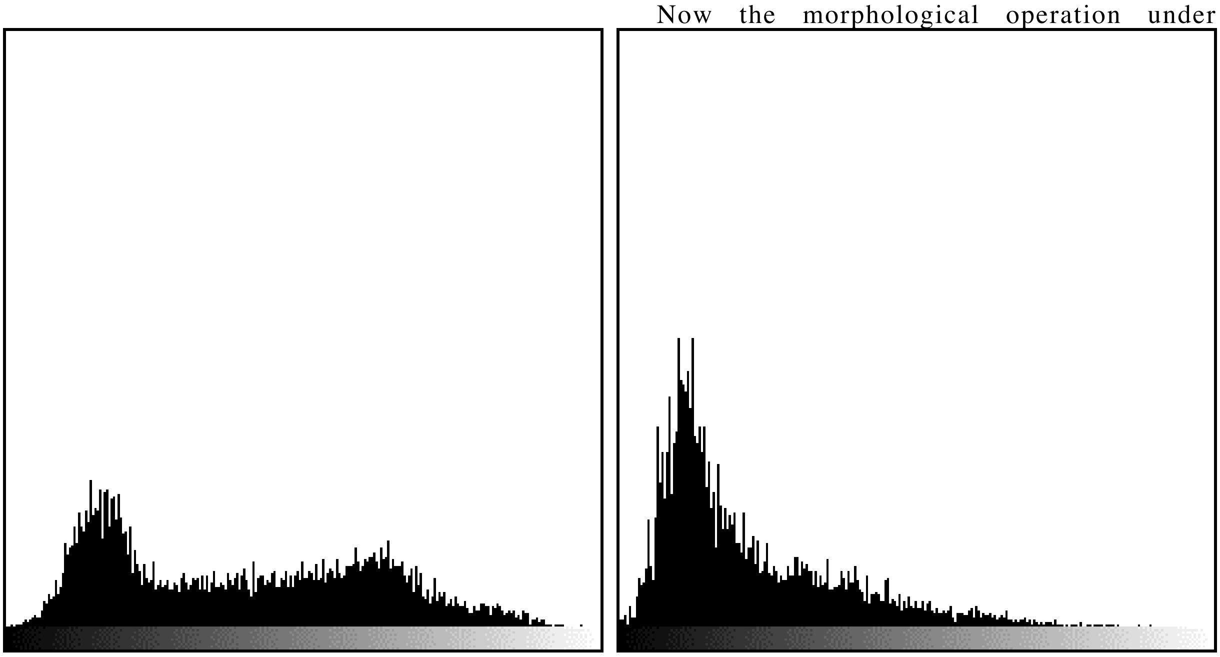
a b

figure 2: Grey-value histogram of figure 1 before (a) and after erosion (b)

New Approach for the fitness function

The new approach for defining a fitness function for evolutionary adaptation of morphological masks is as follows. Firstly the original image was "mixed" to destroy its local structures. This could be done by repeating the exchange of two randomly chosen pixels in the image. In figure 3 the result for this procedure performed on the image in figure 1 can be seen.

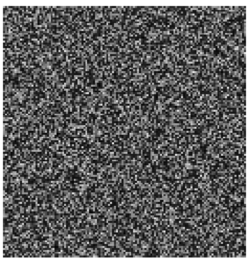

figure 3: Mixed image of figure 1

This image has the same histogram as the original image, hence the influence of morphological operations on the grey-value distribution is comparable.

Now the morphological operation under consideration can be applied to both images, the original one and the mixed one. The volumes of both images (i.e. the sum of all grey-values) can be compared and the optimization goal is to maximize the difference of both volumina. This ensures the structure describing ability of the morphological mask under consideration because it best matches the inherent structure of the original image and rejects the noisy structure of the mixed image.

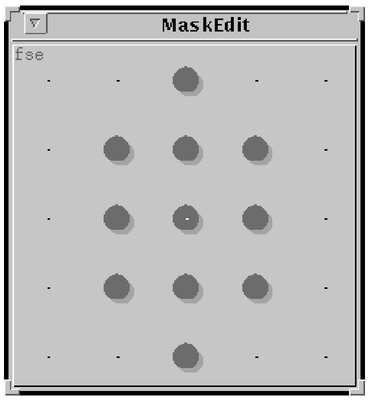

figure 4: Best adapted mask for figure 1

Results

Some experiments were made using an easy implementation of genetic algorithms [5] to adapt a best matching morphological mask for erosion of the original image. The coding of a mask is straightforward by putting a 1 into the genom for every mask position set and a 0 for all others. The genetic adaptation was performed by repeated execution of the basic steps marriage by roulette rule, two-point crossover, mutation and n-best-selection. For the treatment of the textile in figure 1 a population of 7 parent genoms and 20 children genoms were used, the adaptation went over 70 to 200 generations due to different random initializations of the population, and a mask size of 5x5 was used. The result is shown in
figure 4.

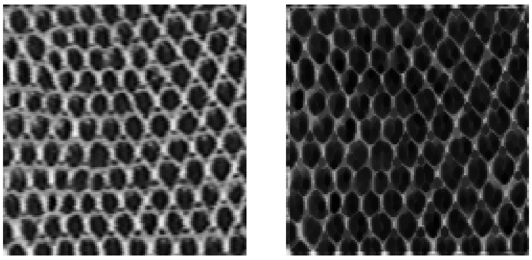

figure 5: Erosion of figure 1 by the mask of figure 4

Similar results could be achieved for other textiles. The methodology proposed failed for adapting masks on non-regular textures like the surface of castings, but this seems to overrun the application area of the proposed approach which is based on the occurrence of local structures.
For the reduction of calculation time for the new fitness function a theoretical assumption about the histogram of the processed scattered image could be made. In the Appendix is the evaluation of a equation for the estimation of the histogram of the scattered image after performing an erosion with a mask with m points set.

Conclusion

A new approach for defining a fitness function for genetic algorithm usable for the adaptation of morphological masks in morphological operation sequences has been introduced. Without this approach it is not possible to use genetic algorithms in grey-scale morphology because of the unpredictable influence of morphological operations on the grey-value distribution in the image function. The example given shows the ability of this approach for finding masks which are able to represent a disturbed pattern in a textured image. An expansion of this approach for finding morphological operations sequences, that perform high-level image processing tasks, is currently under study.

Literature

[1] J. Serra, „Image Analysis and Mathematical Morphology“, Academic Press, London, 1982

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[3] J. Bala, H. Wechsler, „Shape analysis using genetic algorithms“, Pattern Recognition Letters, Vol. 14, No. 12, December 1993, p. 965-973

[4] N. Harvey, S. Marshall, „Using genetic algorithms in the design of morphological filters“, in: J. Serra, P. Soille, „Mathematical Morphology and ist applications to image processing“, Kluwer Academic Publishers, 1994, p. 53-59

[5] E. Schöneburg, F. Heinzmann, S. Feddersen, „Genetische Algorithmen und Evolutionsstrategien“, Addison-Wesley, 1994

Appendix

Approximation of the histogram of the eroded scattered image

Assume the grey-values are randomly distributed inside the image area and there are m positions randomly taken. This should be the points the morphological mask uses for comparison, i.e. for taken the minimum of them. Then we choose the lowest grey-value of them. The question is what is the probability of the occurrence of a special grey-value g.
Collecting this probabilities in an histogram we get an approximation of the histogram of an eroded mixed image.

Consider figure figure 6. There all G pixel are sorted by increasing order of their grey-values. There will be chains of pixels with equal grey-value. The position of the first occurrence of a grey-value $g_1 \geq g$ ($g_1=256$ is chosen if there are no $g1 \geq g$ in the image) is indicated by p_g, for convenience p_{256} is set to G+1.

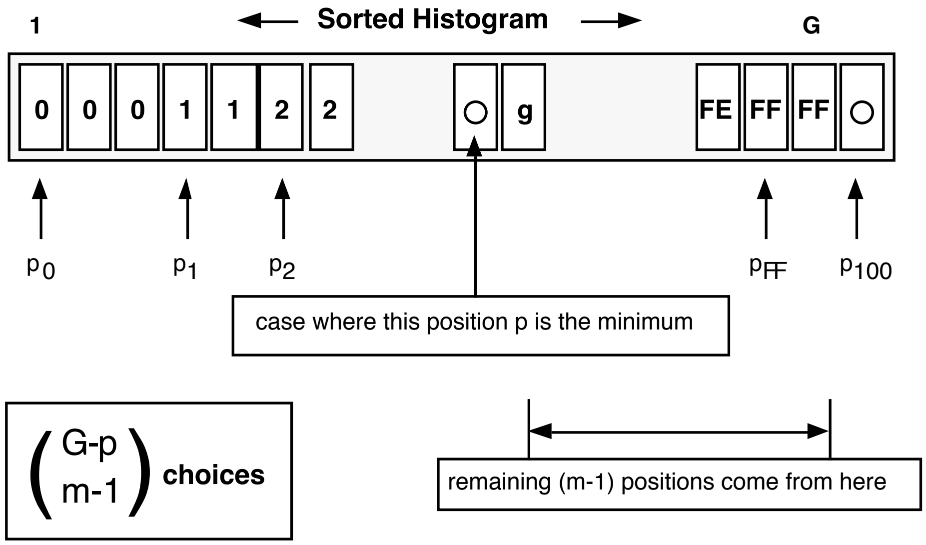

figure 6: Estimation of new histogram

Now assume there are m positions selected from this collection and the first position of it is p. Then the minimum grey-value of this positions is the grey-value at position p. Altogether there are m-1 positions remaining to be selected from the last G-p positions in the collection. This results in the count of choices where p is the position of the minimum grey-value to be:

$$\text{(A-1)} \qquad \binom{G-p}{m-1}.$$

To obtain a formula for the frequency of a certain grey-value g we have to sum up the appropriate terms for all positions p where p refers to grey-value g:

$$\text{(A-2)} \qquad h_A(g;m,G) = \frac{1}{G} \sum_{p=p_g}^{p_{g+1}} \binom{G-p}{m-1}$$

where the convention is used to set

$$\binom{n}{m} = 0 \qquad \text{for } n<m$$

and

$$\sum_{i=n}^{m} f(i) = 0 \qquad \text{if } n>m.$$

$g_{256} = G + 1$ was set to ensure the validity of formula (A-2) for the largest grey-value.

This formula was used to estimate the histogram of a mixed image after erosion to prevent the genetic algorithm from performing the erosions on the mixed image over and over again. The quality of the formula was tested by eroding sample images. The results are of very good quality (see figure 7).

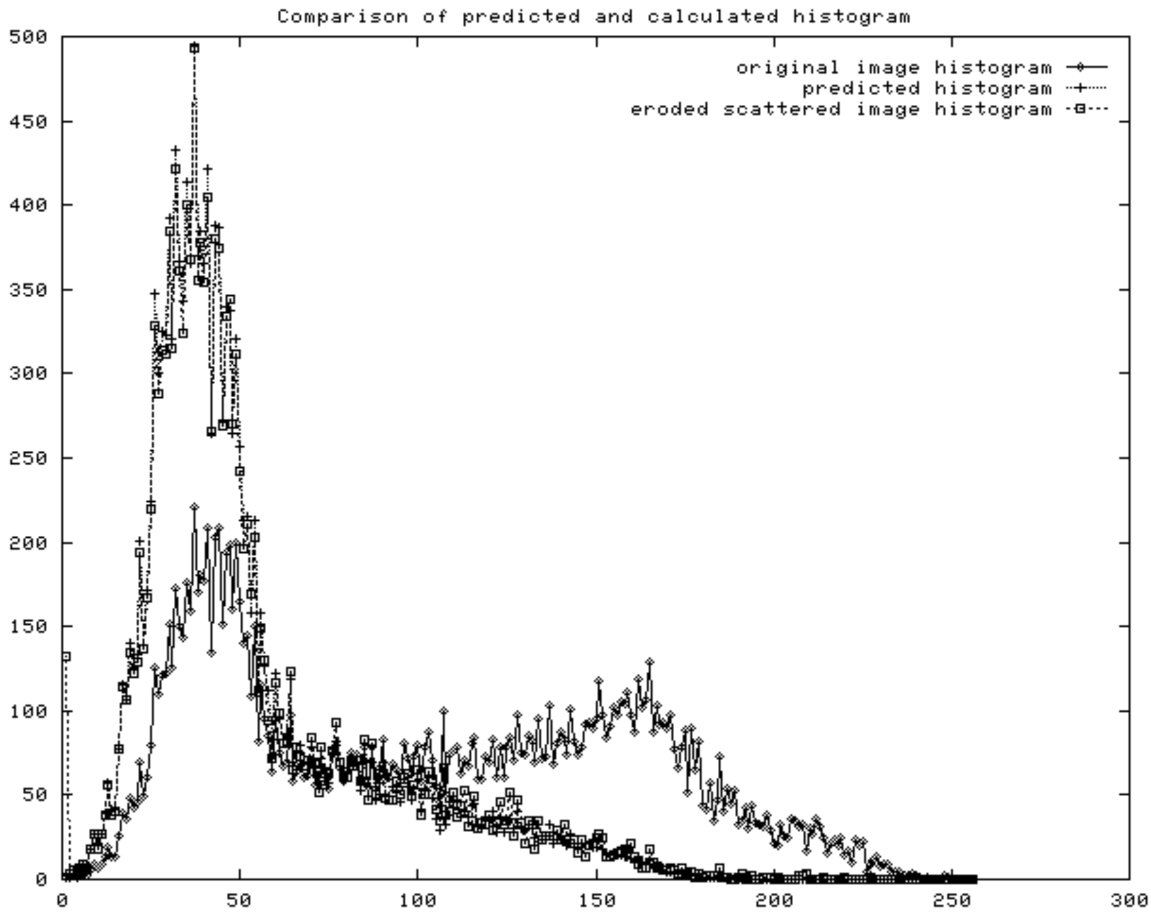

figure 7: Prediction of histogram of eroded image