Fuzzy Color Morphology based on the Fuzzy Integral

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Abstract

In the theoretical framework of Mathematical Morphology a plenty set of binary and grayvalue image processing operations have been developed and successfully implemented. The basic element for the definition of a morphological operation is the existence of an ordering scheme. Due to the vectorial nature of color information the extension of morphological operations for the treatment of color images is not trivial. The paper presents a reduced morphology based on the utilization of the fuzzy integral that can be used with color images. In this case the determination of the fuzzy measures in the fuzzy integral allows the introduction of the new directed morphological operations, which are also presented in this paper.

1. Introduction

Mathematical Morphology is a well-known discipline for the analysis of spatial structures which has found an extended application in image processing. The extension of grayscale morphology, which have been successfully used in multiple applications, to color morphology has very few examples in the literature.

In spite of the uncertainty present in the definition of color, only one work [7] has till now considered the introduction of fuzzy concepts on the definition of such a morphology. The present paper complements this lack by introducing a new fuzzy color morphology based on the concept of aggregation operators as similarity measures presented in [2]. The fuzzy integral is used as aggregation operator for solving the ranking operation on three-dimensional color vectors. Thus a reduced morphology is obtained. A very novel aspect of the here presented morphology is the possibility to define a new class of operations, which can be qualified as directed.

The obtained morphology has been applied on both synthetic and real application images with successful results.

2. Mathematical Morphology

Mathematical Morphology is a well-known discipline for the analysis of spatial structures, which has found an extended application in image processing [9][10]. Under mathematical morphology for image processing a set of operations for different image-to-image transformations by means of a structuring element can be considered. This element, whose dimensions are normally smaller than the image being treated, acts as a structural probe for the suppression, enhancement or preservation of certain elements of the treated image [10].

2.1 From Binary to Grayvalue Morphology

Two basic operations underlay mathematical morphology: dilation and erosion. Up to these basic operations a more or less complex universe of operators can be derived for the resolution of more or less complex tasks.

The original definition of dilation and erosion for image processing was made for binary images. Taking a binary image I, that is, a two-dimensional discrete function on a finite domain ZxZ whose pixels can have values in the discrete domain [0,1], the binary dilation of I by a structuring element S (symbolize with $\oplus_B$) can be defined as [8]:

$$I \oplus_B S = \bigcup_{i \in I} i + S \qquad (1)$$

and its counter part, the binary erosion ($\otimes_B$), as:

$$I \otimes_B S = \bigcap_{i \in I} i + S \qquad (2)$$

The usefulness of binary morphology was limited because of the nature of these images. As a consequence and in order to allow the processing of grayvalue images some generalizations of binary morphology surged: grayscale morphologies. The mathematical definition of both operations for a grayvalue image I is respectively:

$$(I \oplus S)(z) = \max_{z=i+s}\left\{ f(z) \,\middle|\, i \in I, s \in S \right\} \qquad (3)$$

$$(I \otimes S)(z) = \min_{z=i+s}\left\{ f(z) \,\middle|\, i \in I, s \in S \right\} \qquad (4)$$

For the sake of understanding dilation and erosion could be intuitively explained like the following. A structuring element S, which normally adopts the form of an image mask, is rolled over an image I. The value of the pixel within I under the center of S is substituted by the maximum, in case of a dilation, of the pixel values covered by S. In case of an erosion the minimum is taken. This operation is repeated for all the pixels in the image I. In this way a local operation, pixel per pixel, has a global effect on the general aspect of the treated image.

In the last time some works have been presented on the fuzzy generalization of grayscale morphology. These generalizations considered some degree of fuzziness for the treatment of uncertainty. For that purpose many fuzzy concepts have been used: T- and S-norms, the extension principle, fuzzy integral, fuzzy subsethood and OWAs [8]. Only one of these generalizations, presented in [7], considers the problem of allowing the morphological processing of color images.

At this point is worthwhile to remember the key ideas of Serra [9] for the generalization of mathematical morphology to other types of image data [8]:

- the existence of an order relationship,
- the existence of a supremum or infimum pertaining to that order,
- and the possibility of admitting an infinity of operands.

2.2 Color Morphology

Two basic problems arise when trying to conceive a mathematical morphology for color images. On the one hand the multidimensional representation of color. Most of image acquisition devices use a three- or even four-dimensional space for the representation of color. Thus the establishment of a maximum or minimum on this signal is much more complex than just applying a simple comparison between the pixels in the subset of a grayvalue image. Different strategies of multivariate ranking have been proposed for solving this problem.

On the other hand color definition is a subjective, complex and uncertain matter. Not only different features of color vision in human beings [1], including e.g. color constancy in front of changing illumination conditions or hue mutual contrast, supports this affirmation, but also multiple physiological facts. These range from the impairment to distinguish between green and red to little individual dependant differences in the sensitivity curves of the cones in the retina [1]. Thus it seems to be logical that a generalized color morphology considers some fuzzy concept.

Beyond the key ideas of Serra [9] for a generalized morphology, additional requirements were considered to be important for the establishment of morphological operations on color images in [7]:

- No new color values should be introduced in the treated image.
- Dilation should be defined as an increasing operation, while erosion as a decreasing one. That is, when operating with a dilation (erosion) on an image no of its pixel values should be greater (less) than the value former to the operation.
- Through the consecutive appliance of two masks the same result should be obtained as that one obtained through the appliance of a sole mask resulting from their binary dilation.

$$(I \oplus S_1) \oplus S_2 = I \oplus (S_1 \oplus_B S_2) \qquad (5)$$

- A generalized color operation should become a grayscale one when being applied to grayvalue images.
- The operation should present convergence.

Thus not all multivariate ordering principles can lead to a color morphology. Three types of color morphologies were established in [7]: marginal, reduced and Pareto-morphologies. Marginal morphologies consider applying a one-dimensional operation on each of the color channels. Reduced morphologies use a scalar function on the components of the color vector, whose result is employed for the ranking operation. Pareto morphologies use a partial ordering scheme of ranking, i.e. the iterative selection of a subset on the initial point set of discussion, and the relaxation of maximum through a concept related to the Pareto set.

Few color morphologies have been introduced till now [7]. [3] presented a marginal morphology adequate for the processing of label coded images. A reduced morphology was explored in [4]. Finally in [7] Fuzzy Pareto Morphology for color images was presented. In spite of the tight relationship between color and fuzziness, this one has been till now the only attempt to incorporate any fuzzy concept to a color morphology.

Taking as inspiration the work in [2] a new fuzzy color morphology based on reduced ranking is developed in the present paper. The concepts underlying this work are the usage of the fuzzy integral as reducing function, and the relationship between aggregation operators and similarity measures.

3. Aggregation as similarity

In [2] a common theoretical framework for aggregation operators in the context of fuzzy systems is presented. This work states that aggregation operators can be used to measure distances in metric spaces, where an intrinsic relationship exists between the aggregation operator and the distance function being used.

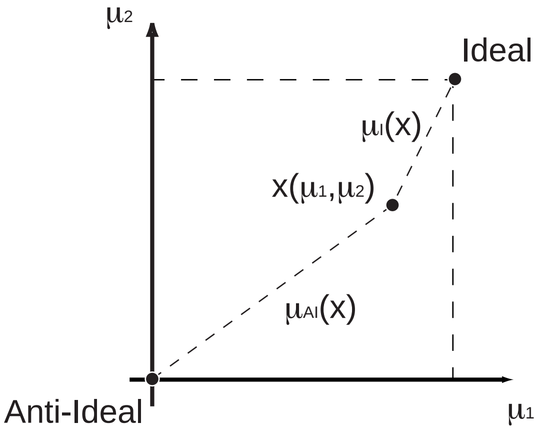


Figure 1: Representation of a fuzzy set x in a two-dimensional space and the corresponding membership functions respect to the sets Ideal and Anti-Ideal.

Up to the utilization of aggregation operators in Fuzzy System Theory to establish the membership function of the output variable a semantical interpretation for this operation is found. The result can be seen to measure the similarity to prototype elements of the output set, which receive the name of Ideals [2].

In this space the Ideal is represented by the point where all the features have a maximum value, which is 1 in the case of normalized features, and its counterpart, which receive the name of Anti-Ideal, by the origin (see figure 1). As a consequence the membership functions of the Ideal (μ_I) and the Anti-Ideal (μ_{AI}) are considered to be [2]:

$$\mu_I(x) = 1 - d(I, x) \tag{6}$$

$$\mu_{AI}(x) = d(0, x) \tag{7}$$

Since these memberships can be calculated through aggregation operators, with the only condition of being monotonic, and having a value between 0 and 1, following relationships can be set:

$$d(x, I) = 1 - M(x) \tag{8}$$

$$d(0, x) = N(x) \tag{9}$$

where M and N are arbitrary aggregation operators for fuzzy AND and OR respectively. For theorem, demonstration and deeper explanation the mentioned work is referred [2].

4. Fuzzy Integral as Aggregation Operator

The question arises if the Fuzzy Integral, as operator with positive properties for aggregation and unique one to allow the expression of interactions between integrands [6], could be employed under consideration of the theoretical framework elucidated in the former section.

4.1 Brief introduction to the Fuzzy Integral

The Theory of Fuzzy Measures was built on the conclusions made in Sugeno's pioneering work [11]. In an analogous way as fuzzy logic extended classical logic by adding subjectivity to the reason process, fuzzy measures consider the extension of classical measure theory with the introduction of subjectivity. In order to satisfy this attribute, classical measures (i.e. probability measures) are extended by relaxing their additivity axiom. As a consequence fuzzy measures include probability, possibility, necessity, belief and fuzzy-λ measures. The establishment of a new kind of measures led to the establishment of a new integral, the fuzzy integral.
The fuzzy integral was introduced in order to simulate the process of multiattribute fusion undertaken in the mind [11]. The basic idea of this approach is that when fusing information coming from different sources, the human being undertakes a subjective weighting of the different criteria or factors, and aggregates them in a non-linear fashion. Moreover the fuzzy integral considers the aggregation of fuzzified data and constitutes a theoretical framework where other aggregation operators as minimum, maximum, average, median, weighted sum, OWA, weighted minimum or weighted maximum are hold [6].

There are basically two types of fuzzy integral known as Sugeno's and Choquet's Fuzzy Integral. The difference between them can be found in the used operators, what in the theory of fuzzy sets receives the name of T- and S-norms. While the Sugeno's Fuzzy Integral (S_μ) uses norms of the type maximum and minimum,

$$S_\mu[h_1(x_1),\dots,h_n(x_n)] = \bigvee_{i=1}^{n}[h_{(i)}(x_i) \wedge \mu(A_{(i)})] \tag{10}$$

the Choquet's Fuzzy Integral (C_μ) was introduced as an extension in the framework of Fuzzy Measures Theory of the classical Lagrange's Integral.

$$C_\mu[h_1(x_1),\dots,h_n(x_n)] = \sum_{i=1}^{n}\left\{\left[h_{(i)}(x_i) - h_{(i-1)}(x_{i-1})\right]\mu(A_{(i)})\right\} \tag{11}$$

For a deeper description of the expressions and the calculation details see [5]. A third type of fuzzy integrals is the so-called Weber's Fuzzy Integral, a generalization of Sugeno's and Choquet's Fuzzy Integrals that allows the employment of different T- and S-norms.

Concerning the properties of the fuzzy integrals as aggregation operators, whose full listing would unnecessary extend the here presented paper, only two of them will be mentioned. Each fuzzy integral is monotonic with respect both to the integrand and the fuzzy measure [5]. Moreover they deliver a value between 0 and 1 when being used with normalized integrands and fuzzy densities. Taking these two properties into consideration there is no impairment for the utilization of the fuzzy integral in the theoretical framework presented in the former section.

4.2 Color Morphology based on the Fuzzy Integral

The intuitive idea behind the establishment of such a new morphology is that when operating with a grayscale morphology the maximum and minimum of the ranking scheme are known, i.e. the pixel values of the images considered range from 0 to g_{max}. Normally the maximum value is the color white and the minimum the black one. The same could be considered on color images, where these two colors could represent respectively the Ideal and the Anti-Ideal of the theoretical framework of aggregation operators as similarity measures. Furthermore a very important property of the new fuzzy color morphology, derived from the usage of the fuzzy integral, would be that the operations dilation and erosion could be defined as directed ones. By directed is meant that pixels with certain proximity to the Ideal (or the Anti-Ideal) would be favored (or not) in the ranking result through the values of the fuzzy densities. In this way one could speak of a red-dilation, a dilation where the red tones would be favored, or a blue-erosion.

For the purpose just described it should be found a pair of fuzzy integrals to be used in the calculation of the distance to the Ideal and the Anti-Ideal taking the place of M and N in expressions (8) and (9). Practical work proved that a good option to calculate this function is the usage of a fuzzy measure for the Ideal and of its dual one for the Anti-Ideal. The relationship between dual fuzzy measures (μ, μ^*) is [6]:

$$\mu^*(\cdot) = 1 - \mu(\cdot^c) \tag{12}$$

Thus the distance to the Ideal can be calculated through the fuzzy integral with a fuzzy measure, which favors some values in front of others and allows this way directed morphological operations, and the distance to the Anti-Ideal through the fuzzy integral with the dual fuzzy measure. In this way the new fuzzy color morphology can be classified as reduced, where the function used for that purpose is the fuzzy integral.

For the calculation of the dilation the pixel under the mask with a minimal distance to the Ideal and maximal distance to the Anti-Ideal will be declared as the winner of the ranking operation. In case of an erosion the minimal distance to the Anti-Ideal and the maximal to the Ideal will be considered in an analogous way. In case of more than one winner the pixel under the center of the mask will be considered, and if this one is not among them the first one beginning from the top left corner to the bottom right one.

Moreover the fuzzy color morphology defined as formerly described satisfy all the requirements detailed in section 1.2 for a color morphology to be considered as such one.

5. Results of Application

The here presented fuzzy morphology was employed on different images in order to show its positive features for practical applications. In all cases the Sugeno's Fuzzy Integral was used. Firstly it was used on an 8x8 image, where the color of the pixels was randomly established (see figure 2).

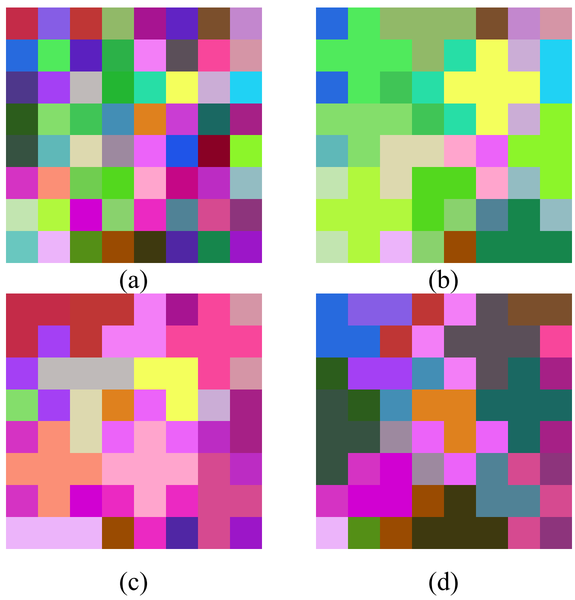
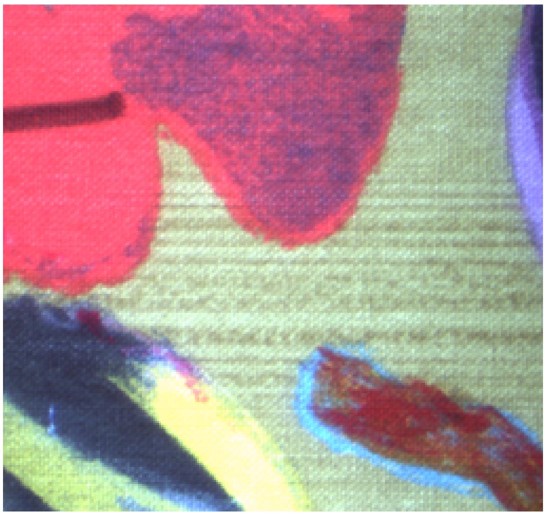


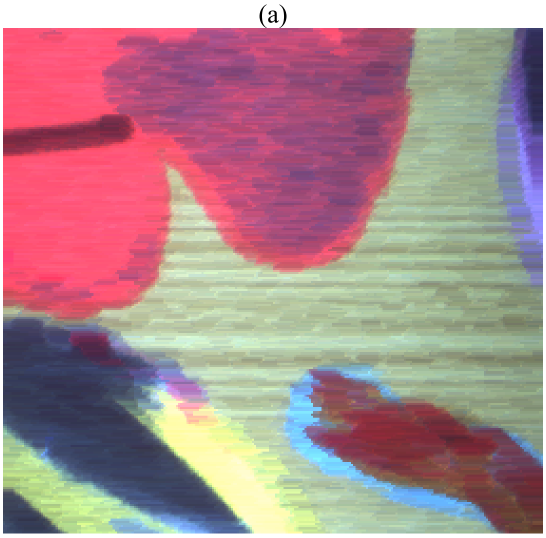
Figure 2: Demonstration of the here presented fuzzy color morphology on hand of an 8x8-image (a), where the color pixel values were randomly set (original images in color). Different directed operations: green-Dilation (b), red-Dilation, and green-Erosion (see explanation in text).

The different operations act as expected. In this way dilation operations (see figs. 2b, 2c and 2d) favor the color expressed by the fuzzy measures. A very interesting effect is the prevalence of the operation type over the direction been set by the fuzzy densities. For instance in an erosion operation (see fig. 2d) pixels of the color fixed by the fuzzy densities are eliminated, except those dark tones of the color. In the dilation operation pixel with hell tones remain, independent of the operation direction.

Moreover the new fuzzy color morphology was used in a practical application, namely the detection of faults on different textiles. In this case the goal was to enhance the faults in order to make easier their posterior characterization (see fig. 3 and 4).



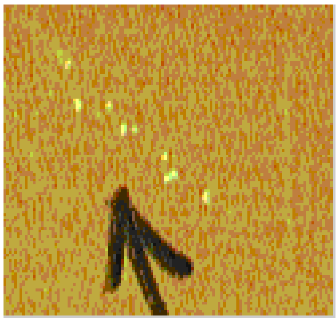
(a)



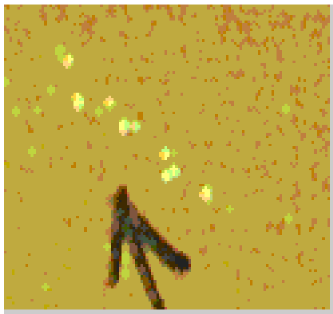
(b)

Figure 3: Enhancement of the fault in a textile provoked by a movement of the ink on the substrate. (a) Original image. (b) Result obtained through a directed erosion with a linear mask. (Original images in color).

As it can be observed in figures 3b and 4b the obtained results were satisfactory.



(a)



(b)

Figure 4: Enhancement of a fault in a textile produced by the presence of dust during the printing process. (a) Original image. (b) Result obtained through a directed dilation with a cross mask. (Original images in color).

6. Conclusions

A new fuzzy color morphology of type reduced was presented in this paper. The fuzzy integral is used as ranking function, where the establishment of the fuzzy densities offers the possibility to define directed operations. Some theoretical aspects of the calculation scheme rest open:

A. The exactly definition of the metric being used.
B. The foundation for the usage of pairs of dual fuzzy measures.
C. The characterization of the new morphology for image processing, and particularly of the different operations for different types of fuzzy integral.

These aspects will be objects of future research works.

7. References

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