Fuzzy–Subsethood based Color Image Processing

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Abstract

This paper presents the exploitation of the concept of fuzzy–subsethood for defining a new class of color image processing operations. By considering a color value as a fuzzy set, the calculus of fuzzy-subsethood becomes applicable to color images. From this, a simple color threshold operation can be defined, which gives a grayvalue image from the degrees of subsethood. Also, mathematical morphology can be extended to color images this way. For doing so, the important class of Pareto–Morphologies is introduced, and fuzzy–subsethood is used to define a new ranking scheme for multivariate data. This gives the Fuzzy–Pareto–Morphology (FPM), which fulfills basic requirements for a generalized morphology. An extension of the difference image to color images and an operation given by an intermediate result of FPM, the M-image, are presented as new operations. Some examples of frameworks, which employs the newly defined operations, are given. It comes out, that fuzzy–subsethood allows for the definition of a new class of comprehensive color image processing operations.

1 Introduction

Color image processing is of essential importance in order to increase robustness, versatility and reliability of technical vision systems. As exemplified by human perception abilities, color is more than a simply "add-on" to grayscale images. Modern computing equipments with a major improvement of its calculation and storing capabilities allows for color image processing to become a nowadays state–of–the–art.

Two questions are considered to be of basic importance for color image processing, the question of color representation, which is strongly related to color spaces, and the question of appropriate color image processing operations. While investigations on the first question resulted in a wide variability of technical, psychological or theoretical important color models (as HSI, RGB, CMYK, Lab, but there are much more…), research on the second question has been performed in a more restricted manner. Some basic problems related to color image processing are: multivariate nature of color data, which complicates the extension of some grayscale operations to color images (e.g. convolution, mathematical morphology); and the dual nature of human (or mammalian) color perception sensitiveness: being highly sensitive to smallest "color artifacts," and being highly insensitive for luminiscance variations within images (e.g. under varying lightning conditions) at once.

This complicates and restricts possible definitions for versatile operations on color images. Neither Laplacian, Sobel, Dilation nor Thresholding found suitable counterparts in color spaces so far. To overcome this, in this paper a means is given for designing a new class of such operations, employing the calculus of fuzzy–subsethood.

This paper is organized as follows: in section 2, the concept of fuzzy–subsethood is shortly recalled. Then, section 3 presents the new operations Color Thresholding (section 3.1) and Color Morphology (section 3.2). In order to define the new generalized morphology, a broader class of multivariate morphologies, the Pareto–Morphologies, has to be introduced. The Fuzzy–Pareto–Morphology is proposed in this section, and also some accompanying operations. Some demonstration examples, which prototype possible frameworks for applying the new operations, are presented along with the defintions. The paper ends with the acknowledgment and the reference.

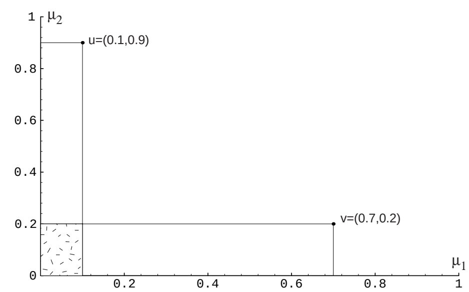


Figure 1: Fuzzy sets as points in the unit square.

2 Fuzzy subsethood

Normally, a fuzzy set is given by the membership degrees of its elements

$$M = (\mu_1, \mu_2, \ldots, \mu_n)$$

According to the fuzzy set approach of Kosko [7], fuzzy sets could be considered as points in the n-dimensional unit square (or unit cube) by using the membership degrees as coordinates. If the parallels to the coordinate axis are drawn, a hyperrectangle is constructed this way. Hence, each fuzzy set corresponds to a hyperrectangle in the unit square. Fig. 1 shows 2 two-dimensional fuzzy sets. This approach gives the way for a redefinition of the term subsethood. Initially, a fuzzy set A was considered as a subset of fuzzy set B, when for all membership values $a_i \leq b_i$ is fulfilled [12]. Kosko extended the concept to degrees of subsethood. This degree is derived geometrically from the assigned hyperrectangles. It equals the ratio of the volume of the intersection of both hyperrectangles to the volume of one hyperrectangle. Thus, even the whole set is a subset of each of its subsets to a certain degree[1].

[1] This was considered as fuzzy foundation of probability by Kosko.

3 Fuzzy subsethood based color image processing

3.1 Color thresholds

The simplest operation, which can be defined by using fuzzy subsethood, is to take a color value as three-dimensional fuzzy set, and derive the degree of subsethood of each image pixels' color value. Thereby, each color value is considered as a fuzzy set, too. The degree as a value from $[0,1]$ is linearly mapped onto a grayvalue from $\{0,\ldots,g_{max}\}$. More formally, if (c_x, c_y, c_z) is the choosen color threshold, and an image pixel position has the color value (p_x, p_y, p_z) for a choosen color model, then the same position in the result image gets the new grayvalue

$$g_{new} = g_{max} \frac{\min(c_x, p_x)\min(c_y, p_y)\min(c_z, p_z)}{c_x c_y c_z}.$$

Clearly, each component of the color threshold must be different from Zero.

The result of this "color thresholding" is a grayvalue image. Despite of its simple nature, color thresholding allows for the design of useful operation, for example highly sensitive color texture filters.

Consider the color textile example given in fig. 2. The HSI representation of this image is thresholded twice, with color threshold (0,200,100) and with color threshold (0,100,80) (the color thresholds are taken from $\{0,\ldots,g_{max}\}$ and rescaled to $[0,1]$). The result of the second thresholding is subtracted from the result of the first thresholding by pixelwise subtraction of the grayvalues. The resulting image is given in fig. 3 (b). For comparison, the standard grayvalue transformation of color images is given in fig. 3 (a)[2]. As can be seen from the threshold difference image (b), the seemingly homogenuous background texture of the textile reveals its pecularities. These are due to surface faults, but also due to errors in the shading correction of the used scanning device.

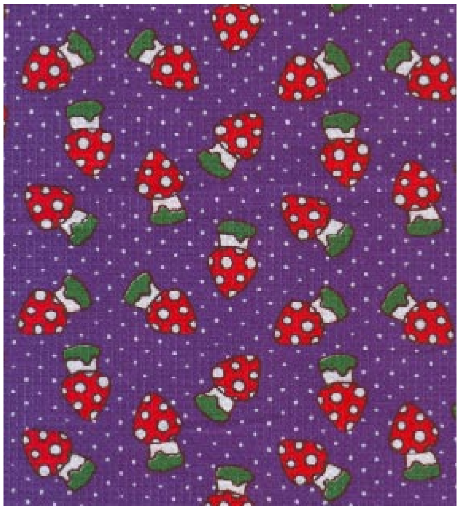
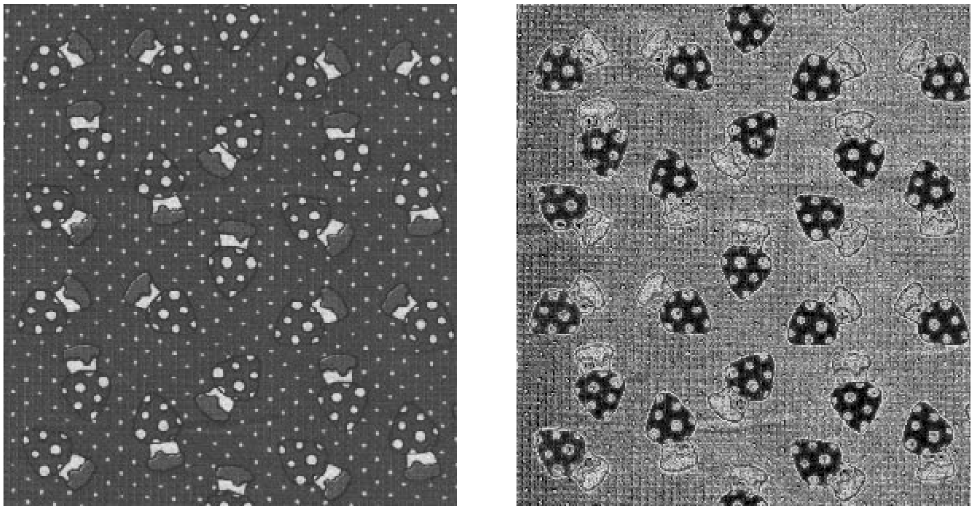


Figure 2: Original color textile image.

[2]The original color plates can be found at *http://vision.fhg.de/ipk/koeppen/images/nfs99*.



(a) (b)

Figure 3: Comparison between standard grayvalue conversion (a) and the difference of two color–thresholdings of fig. 2 (b) (see text for details). The latter one reveals some anomalies of the background texture.

3.2 Pareto–Morphology

3.2.1 Generalized morphologies

Each generalized morphology will be based on its definition of two basic operations: dilation and erosion. As suggested by Serra [8], there should be three key ideas, based on which a generalized dilation is defined[3]: an idea of ranking due to a sort order; an idea of a supremum due to this ranking; and the possibility of admitting an infinity of operands. The issues related to generalized morphologies, especially in the context of fuzzy logic, have been intensively discussed in [2] [3] [6] [4] and [11]. Consider also [8] [9] and [10] for a comprehensive introduction into the field and the applications of mathematical morphology for image processing.

3.2.2 Multivariate ranking

In order to fulfill these requirement for multivariate data, the concept of ranking n values should be extended to the ranking of n vectors. In [1], multivariate ordering principles were classified into four categories: marginal ordering, reduced ordering, partial ordering and conditional ordering. Only marginal ordering and reduced ordering can be used in order to define the set function of a generalized dilation. An example for a generalized dilation based on marginal ordering is given by applying a one-dimensional set function P component-wise:

$$P \circ \{(p_x, p_y, p_z)\} = (P \circ \{p_x\}, P \circ \{p_y\}, P \circ \{p_z\}).$$

[3]Generally, when the definition of a dilation is fixed, the erosion is defined as the complementary operation.

However, marginal ordering would give new color values within the result image, thus possibly producing “color artifacts.”

By reduced ordering, a scalar parameter function is computed from the vector components of each color value within M. The ranking is performed according to the resulting scalar values. Generalized morphologies can be designed based on reduced ordering. This has been intensively discussed in [4]. The comparison of these morphologies with the here proposed Fuzzy–Pareto-Morphology is currently investigated.

Multivariate ranking is important for solving multi-objective optimization problems. Recent approaches here made use of the concept of the Pareto set [5]. If two vectors $\vec{a}$ and $\vec{b}$ are to be compared, it is said that $\vec{a}$ dominates $\vec{b}$, when each component of $\vec{a}$ is at least as large than the corresponding component of $\vec{b}$, and at least one component is larger:

$$\vec{a} \ >_D \ \vec{b} \longleftrightarrow \forall_i(a_i \geq b_i) \wedge \exists_k(a_k > b_k).$$

The subset of all vectors, which are not dominated by any other vector, is the Pareto set (also Pareto front). From this, we define the subset operator **P**, which assigns the Pareto set to a set of vectors.

The Pareto set describes the possible solutions of a multi–objective optimization problem. According to the problem statement, every solution, which gives an element of the Pareto set, when its multiple criteria are computed, is optimal in this generalized sense.

3.2.3 Pareto–Morphology

The concept of Pareto sets allows for a new interpretation of the supremum of multivariate data. The operator **P** comprises a natural extension of the max operation. Hence we formulate the following requirement for a generalized (e.g. color) morphology:

The result of the dilation must not be dominated by its original value.

$$\sim \exists_p : p \ >_D \ p \oplus a.$$

A generalized dilation is said to fulfill the Pareto property, if its set function P gives the same result for a neighborhood M, if restricted to $\mathbf{P}(M)$.

Every morphology derived from a generalized dilation with the Pareto property is referred to as Pareto-Morphology.

Reduced ordering gives a Pareto-Morphology, if its parametric function f is monotonic. If not, it could violate the Pareto property[4].

At this point, the important question comes up, whether there exists a Pareto-Morphology, which is not based on reduced ordering. The answer to this important question will be given next.

[4]Consider $f(x,y) = \sin\frac{\pi}{2}x + sin\frac{\pi}{2}y$ and its values $f(1,1) = 2$, $f(2,2) = 0$ and $f(1,3) = 0$. While the Pareto set is given with $\{(2,2),(1,3)\}$, the selected point is (1,1).

3.2.4 Definition of Fuzzy–Pareto–Morphology

Fuzzy–subsethood can be used to define a new ranking scheme, which does not belong to one of the four categories of Barnett [1]. At first, we consider maxmin operations of the kind

$$\arg\max(\min(\vec{f}(p))$$

and

$$\arg\min(\max(\vec{f}(p)).$$

When $\vec{f}$ is derived from fuzzy–subsethood degrees of one p within the other p, the resultant ranking scheme is neither marginal nor reduced ordering. Moreover, it allows for the design of a new generalized dilation. This will lead to the Fuzzy-Pareto-Morphology (FPM).

Thus, we can give the definition of the FPM. If the neighborhood M of a pixel is given by n pixels with color values x_{ij} with $i = 1, \ldots, n$ and $j = 1, 2$ or $j = 1, 2, 3$, then the set operation P is given by:

$$P \equiv \mathrm{argmin}_i \left[\max_{k \neq i} \frac{\prod_j \min(x_{ij}, x_{kj})}{\prod_j x_{ij}} \right]. \tag{1}$$

The dilation of FPM is derived from this set function, which gives the replaced supremum value of a neighborhood M of a pixel.

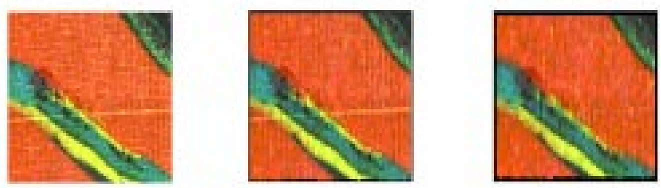


Figure 4: Detection of a thread in a colored textile by filtering it out.

An example of the application of FPM to color images is given in fig. 4. There, the detection of thread faults in a color textile is demonstrated. The structural property of the threads horizontal orientation is used by the FPM. The structuring element is a vertical oriented mask of size 7. If opening, i.e. dilation followed by erosion, is applied with this mask, the thread “vanishes.”

Fig. 5 demonstrates the detection of blots in a colorized texture. From left to right: the original image part[5]; the result of FPM dilation with a structuring element of size 3×3; the result of applying this operation twice; and the result for applying it three times.

3.2.5 Properties of the Fuzzy–Pareto–Morphology

The following properties of the FPM can be verified from its definition.

1. The FPM is a Pareto–Morphology.

[5] The arrow is just for marking the fault, but it is processed as well.

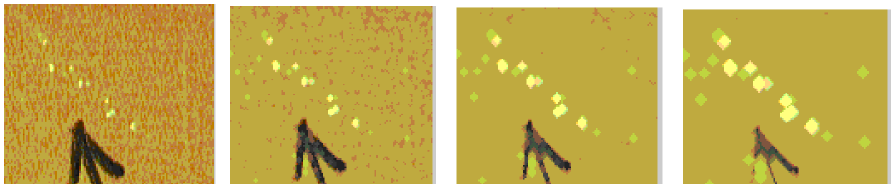


Figure 5: Detection of blots in a colored textile by repetetive application of Fuzzy–Pareto–Morphology.

2. The FPM does not comprise a reduced ordering, except for exactly two points.

3. FPM reduces to standard grayscale morphology, when applied to grayscale images (i.e. color values of dimension 1).

4. But, FPM is not contiguously, i.e. it violates

$$(p \oplus a) \oplus b = p \oplus (a \oplus_B b)$$

for certain values of a, b and neighborhoods p ($\oplus_B$ stands for standard binary morphology).

3.3 Accompanying operations

Within the context of FPM, other operations for color image processing can be designed, too. Two examples will be given in the following:

- A fuzzy color image subtraction operation of two images p_1 and p_2, by which the degree of fuzzy subsethood of the color value of p_1 in the corresponding color value in p_2, multiplied by the original color value, is assigned to each image position.

  This is important for e.g. the definition of the morphological gradient in FPM. In grayscale morphology, the morphological gradient is the difference of dilated and eroded image by the same structuring element. However, simply subtracting two color values would introduce alien color values in the result image. By replacing subtraction with the mutual fuzzy subsethood operation, this could be prevented[6].

  With this operation, edge operators from morphology can be used in applications.

- The selection scheme of equation 1 actually computes values, for which the argument leading to the smallest value is taken as result. However, the lowest value itself can be taken as a grayvalue, and a grayscale image can be constructed this way (the so-called M-image).

These operation may support the processing of color images by the newly proposed FPM. Fig. 6 shows the M-image of the third dilation in fig. 5. The blots are clearly

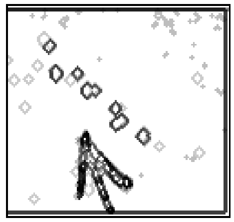


Figure 6: Indication of blots in the colored textile of fig. 5 by the M-image.

indicated.

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