Pareto-dominated Hypervolume Measure: An Alternative Approach to Color Morphology

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Abstract

In this paper, an alternative approach to the non-linear filtering of Mathematical Morphology for color and multi-spectral images based on the Pareto-dominated Hypervolume measure is presented. Parteo-set theory is studied in this context, and among others, successfully implemented in the morphological filtering of color images. The demand to assess the quality of a multi-objective optimization algorithms, in particular, has put forth several kind of measures. One of these measures, the Hypervolume, bases on the Lebesque measure of all points dominated by a set of points and maps a set of Pareto-optimal points to a scalar. By considering the Hypervolume in the color-image domain, it can be shown that this Hypervolume corresponds to another kind of Color Morphology, were each pixel in the filtered image represents the Hypervolume of its set of neighbours in the original color image. In the following, some properties of the Hypervolume, as used as an image-processing filter will be derived, and some potential applications of this approach to Color Morphology will be shown.

1 Introduction

Color image processing is of essential importance in order to increase robustness, versatility and reliability of technical vision systems. Potential application of color image processing are for example object recognition, image understanding, retrieval and compression. Since modern computing equipments with significant improvements of its calculation and storing capabilities are becoming available far and wide, color image processing is becoming a kind of state-of-the-art. However, its fundamentals as for example a realistic color representations in different electronic devices such as digital cameras, monitors and printers, the mapping of different color spaces and the appropriate color-processing filters are still subject of intensive research. Basic problems of color-image-processing filters, that are the focus of our research, are (I) the multi-variate nature of color data that complicates the extension of some gray-scale image filters to the color domain, and (II) the dual nature of human color-perception sensitiveness, that is being highly sensitive to smallest "color artifacts," and being highly insensitive for luminescence variations within images, e.g. under varying lightning conditions at once. In this paper we propose a new concept for handling the multi-variate color-image data that yields to an alternative approach of morphological color-image filtering.

To start with, morphological filtering is an non-linear image-to-image transformation by means of a structural element that acts like a probe sensitive for structural information. Mathematical Morphology in general is based on set theory [9, 10]. Its core definitions are fixed for binary and gray-scale images. The two basic morphological operations are dilatation and erosion that can be used for the definition of more complex filters. Further advanced approaches consider concepts from fuzzy logic in the definition of the dilatation [1] (see e.g. [1, 6, 7, 10]). All these shape-based filters are able to preserve / enhance structural information while suppressing noise or removing clutter. Sine pattern and edge information are often crucial to image understanding, morphological filter operations found a number of challenging applications as for example medical-image analysis, surface-quality inspection and check processing, e.g. [11, 5]. Subsequently, the question arises in which way these powerful binary and gray-scale morphology filters can be generalized and adopted for the processing of multi-variate data in color images.

Requirements for a generalized dilation as an image-to-image transformation, which employs a structuring elements, are still under discussion. As suggested in [9], there should be three key ideas, based on which the dilation is de-

[1] Generally, when the definition of a dilation is fixed, the erosion is defined as the complementary operation.

fined: (1) an idea of ranking due to a sort order, (2) an idea of a supremum due to this ranking, and (3) the possibility of admitting an infinity of operands.

Color is more than a simply “add-on” to gray-scale images, as exemplified by human perception abilities. In addition, there is no unique way to extend the concepts of gray-scale morphology to color images. The fundamental lack of a “natural sort order” of multi-variate data and the numerical differences due to the choice of different color spaces make it hard or even impossible to define something like THE Color Morphology. But it could be expected to transfer a large number of gray-scale morphological techniques to color images. It could be expected to design highly color-specific filters from morphology as well.

Different viewpoints have led so far to the proposal of a number of useful operations for the processing of color images. Among these viewpoints we can find the linear weighted, or scalar, approaches, where a multi-variate channel-intensitiy vector is mapped onto a point by a scalar function that is monotone in each argument.

In this paper, we are studying an intensity-based Color Morphology, with its main difference to other Color Morphologies being the generation of a gray-scale image that cannot be the result of a morphological operation on a gray-scaled version of the color image itself. The formal techniques for achieving this goal come from the field of multi-objective optimization and its related concept of *Pareto dominance*. A consideration of the various Pareto-set-based means and techniques, which have been developed in the past for the study of (continuous) multi-objective optimization problems, leads to the formulation of a number of image-processing operators. A simple example is the generation of a gray-scale image from a color image, where each pixel’s gray-scale represents the number of Pareto-dominating points in the neighborhood of this pixel. Practically this comes out to be an edge operator. However, recent interest has been grown on the use of the so-called *Hypervolume measure* in order to access quality in multi-objective optimization. Given a set of points, the Hypervolume is defined as the *Lebesque measure* of the set of all points that are Pareto-dominated by at least one of the given points. By considering the Hypervolume in the color-image domain, it can be easily seen that this Hypervolume corresponds to another kind of Color Morphology, were each pixel in the filtered image represents the Hypervolume of its set of neighbours in the original color image. In the following, some properties of the Hypervolume, as used as an image-processing filter will be derived, and some potential applications of this approach to Color Morphology will be shown.

The reminder of this paper is organized as follows: Section 2 deals with the fundamentals of Pareto-set theory that are needed for the definition of the Hypervolume detailed in section 3. The following section 4 then gives the options of using the hypervolume as a base for the specification of morphological operation. The paper ends with a conclusion section.

2 Pareto-set Theory

The notion of *Pareto efficiency* originally stems from economics. In the late 19th century, Vilfredo Pareto established a model for the economic stability of a society, where an economy is in a state where one can only become more rich if someone else becomes more poor. In modern terms, given a feasible set of multi-variate values (vectors, points, objectives, decision criteria), each of its elements is considered to be Pareto-efficient, if for any other element having a larger component there is always another component that is smaller. This notion gave raise to the definition of a *Pareto-dominance relation*. For two vectors $a = (a_i)$ and $b = (b_i)$ from R_n it is said that a Pareto-dominates b if and only if

$$a >_D b \leftrightarrow \forall_i (a_i \geq b_i) \wedge \exists_j (a_j > b_j) \tag{1}$$

where $i, j = 1, 2, \cdots, n$. This definition is accompanied by the related definition for minimum dominance, where the “$>$” is replaced by “$<$” and “$\geq$” is replaced by “$\leq$.” To avoid confusion, we will refer to the former one also as *maximum Pareto dominance*, and by *minimum Pareto dominance* to the latter one.

Note that in general not $a >_D a$. There is the extension to the so-called weak dominance, or $a \geq_D b$, if and only if either $a = b$ or $a >_D b$. Also, the (weak) Pareto dominance is transitive, i.e. from $a >_D b$ and $b >_D c$ follows $a >_D c$, but is not given a complete ordering relation, as for two vectors a and b neither $a >_D b$ nor $b >_D a$ may hold (example are the two vectors $(1, 2)$ and $(2, 1)$).

Given a set of vectors $V = v_i$, the term (maximum / minimum) *Pareto-set* refers to its subsets of all elements that are not (maximum / minimum) Pareto-dominated by any other element of the set.

In general, a Pareto-set may give raise to a number of quantitative measurements. By indicating with $P_{max\,/\,min}(S)$ the maximum / minimum Pareto-set of the set S, and by $|M|$ the number of elements of an arbitrary set M, the following measures can be introduced:

- The percentage of non-maximum dominated elements: $r_{max} = |P_{max}(S)|/|S|$.
- The percentage of non-minimum dominated elements: $r_{min} = |P_{min}(S)/|S|$.
- The absolute difference between r_{max} and r_{min}: $r_\delta = |r_{max} - r_{min}|$.

- For any element $s \in S$, the number $n_{pmax/pmin}(s, S)$ of elements of S that maximum / minimum-Pareto dominate s.
- For any element $s \in S$, the number $n_{amax/amin}(s, S)$ of elements of S that are maximum / minimum Pareto-dominated by s[2].
- The largest and smallest values of $n_{pmax/pmin}(s, S)$ and $n_{amax/amin}(s, S)$ among all $s \in S$.

The list could be easily continued.

The Pareto-dominance relation allows for an ordering of a set S of vectors by assigning rank values to each element. This is the so-called non-dominated sorting, and further operations can be based on this ranking. All elements of $P(S)$ get rank 1 assigned, all elements of $P(S - P(S))$ rank 2 and so on, until the remaining set is empty. The disadvantage of this way of ordering is that elements that are different, but of the same rank are indistinguishable, and this ranking cannot be further refined.

Ranking in the multi-variate domain is not restricted to the Pareto-dominance relation. A less rough way of ordering is the so-called lexmin ordering. Here, in case of maximum preference, the element with a smaller largest component would come before an element with a larger largest component. If both element have the same largest component, then comparison is based on the second-largest component, if they are equal on the third-largest and so on. While the Pareto non-dominated sorting usually assigns rank 1 to more than one element, the lexmin ordering only selects one element as firstly ranked (and all that are equal to it, but only these).

Both strategies can be combined, since the first element in the lexmin sorting is also an element of the Pareto-set. So, the lexmin can be used to sort the elements of the non-dominated sorting of the same rank. However, several other secondary ranking assignments have been proposed in the literature, especially methods related to the NSGA-II algorithm [8], an evolutionary algorithm for multi-objective optimization [2].

Each of the proposed methods can be used for the specification of an image-processing operation on color images. If the image is provided in an intensity-based (technical) color space like RGB, then a pixel is a mapping from the image coordinates (x, y) to a tupel (r, g, b), where r indicates its red color component, b its blue component and g its green component. Usually, r, g and b are taken from the integers in $(0, \cdots, 255)$, and the pixel coordinates are integers taken from a rectangularly bounded domain $(0, \cdots, (width - 1)) \times (0, \cdots, (height - 1))$.

[2]The "a" and "p" stand for active and passive dominance in the relation $a >_D b$, i.e. the fact that *a dominates b* (active) or *b is dominated* by *a* (passive).

A masked operator is an image processing operation, which is based on a topology (or neighborhood system) that is assigned to the image pixel coordinate domain. The mask M of a masked operator assigns a subset of the image pixel coordinate domain to each pixel. In the most common form, these are the direct neighbours of the pixel, but other assignments are feasible as well. The mask is also sometimes referred to as structuring element, especially in the discipline of Mathematical Morphology.

A number of masked operators for color images based on Pareto-set analysis can be easily defined, using any of the operations listed in the beginning of this section. Figure 1 gives a few examples for such operations. It is notable that the operators based on the number of dominating neighbours appear to work as edge detectors in the image.

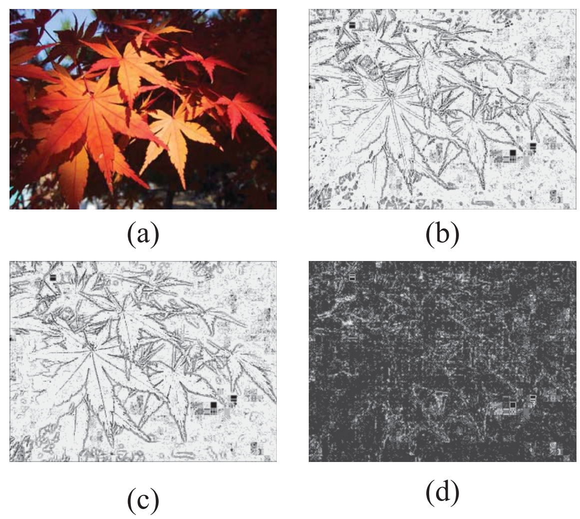

Figure 1. Pareto-set measurements as masked image operators in the 8-neighborhood (8NB): (a) original image; (b) number of points in 8NB dominating central point (CP); (c) number of points in 8NB dominated by CP; (d) size of Pareto-set in 8NB.

3 Hypervolume and its properties

Recently, and in relation to the design of heuristic multi-objective optimization algorithms, interest has grown in the so-called Hypervolume. We can find evolutionary multi-objective optimization algorithms employing the concept of the Hypervolume [12, 3], as well as multi-objective counterparts of the simulated annealing algorithm [4].

The Hypervolume is defined as follows: given a set $V = v_i$ of N vectors from a bounded domain in R_n (the feasible space), the Hypervolume is the Lebesque measure of all vectors (i.e. points in the feasible space) that are Pareto-dominated by at least one of the vectors in V.

If we assume for example the feasible space to have a lower bound of 0 for all components, and target for maximization, than each vector v (or point) in R_n^+ dominates all points within the hyper-cube having v as its outermost corner. The Hypervolume then is the union set of all these hyper-cubes for all vectors in V (see fig. 2).

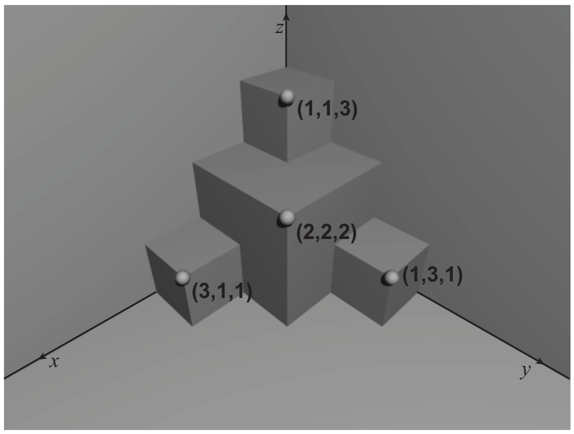

Figure 2. Example for the hypervolume enclosed by the points $(1,1,3),(1,3,1),(3,1,1)$ and $(2,2,2)$. The union of the four cubes has a volume of 11..

4 Hypervolume usage as image-processing operator

4.1 Preliminaries

In gray-scale image processing, an operator is considered to be a dilation if it commutes with the supremum. A similar specification can be given for color dilation, by using the concept of Pareto-dominance. Then, any operation that commutes with the Pareto-set set operator (i.e. the mapping of a set S to $P(S)$) is considered a color dilation. Practically, this has usually been seen as the provision of a selection procedure from the Pareto-set of a color pixels' neighbours (if seen them as three-dimensional vectors (r, g, b)). A simple example is to use as color value at position (x, y) in the processed image the color value of all neighbours of (x, y) (including (x, y)) in the processing image with the maximum sum $r + g + b$ (or one of them if there are more).

Since this will always give a single color value vector, the Pareto-set is composed of this color value, and the operation always commutes with the operation of taking the Pareto-set. However, it has not been considered so far that the operation could also map into the gray-scale domain. Considering the Hypervolume, it is obvious that its value does not change under the addition of dominated points. So, it can be seen as an alternative way of specifying a morphological operation. Since we may consider maximization or minimization, and the Hypervolume of the union and section, we find the following four Hypervolume-related morphological operators:

1. maximum Hypervolume of the union of the vectors dominated by the p_i, and divided by g_{max}^3
2. maximum Hypervolume of the section of the vectors dominated by the p_i, and divided by g_{max}^3
3. minimum Hypervolume of the union of the vectors dominated by the p_i, and divided by g_{max}^3
4. minimum Hypervolume of the section of the vectors dominated by the p_i, and divided by g_{max}^3

and as a first composition the six pairwise absolute differences between these four operations. Examples for all these operations are shown in the figures 3 and 4.

Figure 3. Application of the four hypervolume methods to a color image: 1 maximum union, 2 minimum union, 3 maximum section and 4 minimum section hypervolume.

4.2 Potential applications

As in the binary or gray-scale morphology, starting from the basic operations dilation and erosion, a number of other operators can be defined (opening, closing, thickening, thinning, top-hat transform etc.). This can also be done for the proposed Hypervolume based morphology. Here we are

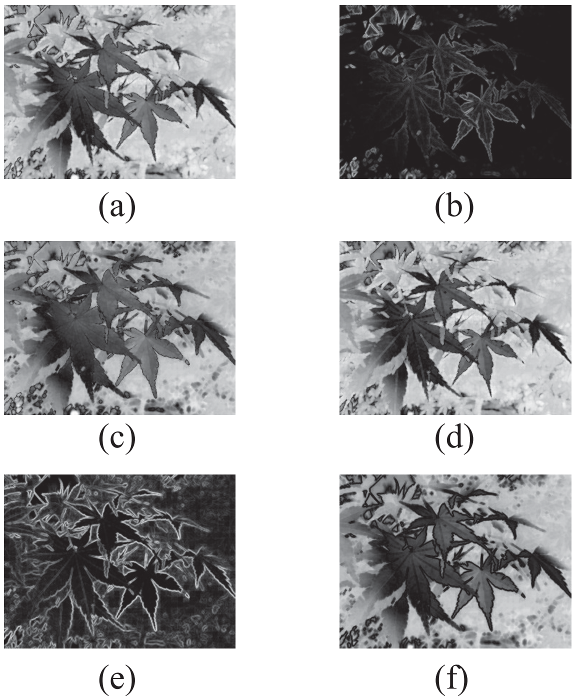

Figure 4. Pixel-wise absolute differences between the gray-values of pairs of hypervolume measures: (a) maximum union and minimum union, (b) maximum union and maximum section, (c) maximum union and minimum section, (d) minimum union and maximum section, (e) minimum union and minimum section, and (f) maximum section and minimum section.

considering the use of such operator for the processing of the scan image of a security document. Figure 5 shows a part of the scan of an ID-card, where some security features are only visible in the UV light. However, UV scan might be to expensive for e.g. small, mobile devices. The right half shows, how the (absolute) difference image between maximum union hypervolume and maximum section hypervolume helps to visualize the hidden security features even from a camera image in the visible light spectrum. The small activations by the small eagle patterns in the blue channel lead to a more or less uniform representation of these areas after the hypervolume processing. With such an approach, for example, it becomes possible to check hidden security features that are difficult to reproduce in low, medium effort counterfeit passports.

At this point we want to emphasis that the approach to color morphology presented in this paper is able to enhance structural information that varies in a very little color range only.

Figure 5. The difference image between maximum union and maximum section hypervolume applied to a scan of a security document visualizes features (the small eagle symbols) that can usually only be seen under UV light.

5 Conclusion

In this paper, we have discussed the opportunities for taking analytical methods of multi-objective optimization to the domain of color image processing. The multi-variate nature of color images allows for a more or less direct translation of Pareto-set theory based measurements to image processing operators. Such operators can account for the local dominance relations among the pixel in the neighborhood of a pixel. Counting dominance relations is one easy way to specify new operators, and some of them come out to be edge operators. One of the more interesting recent tools of multi-objective optimization analysis is the hypervolume, which stands for the Lebesque measure of all points dominated by a given set of points, and is an example for the so-called indicator functions. The hypervolume, if used as an image processing operator, appears to be a morphological operation, since it is commuting with taking the supremum (infimum), meaning the taking of the Pareto-set of a set of points. In contrary to other color morphologies, which now can be characterized as "selection-based", this morphology is intensity-based. The result of the application is a gray-value image, and it was exemplified that such an approach can be much more separatively (due to the higher dimensionality) for small intensity fluctuations in local pixel neighborhood.

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