Fuzzy Image Processing by using Dubois and Prade Fuzzy Norms

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Abstract

This paper presents new image processing operations based on the Dubois and Prade proposal of a fuzzy triangular norm. This definition is recalled, and a computational efficient algorithm is derived for its fast computation. This procedure also gives a comprehensive model for reasoning about the qualitative effects of the Dubois and Prade fuzzy norm on data. The presented image processing operations can be considered fuzzy morphology operations. Due to this fact the application modes for those operations can be judged. Using Dubois and Prade fuzzy morphology for background removal on bankcheck images is demonstrated as an application.

1. Introduction

Triangular norms, which were initially developed for metrics of statistical spaces [6], have recently found a high interest in fuzzy theory [4]. Especially they are used for providing formal definitions of fuzzy set union and intersection operations. Besides of the standard triangular norms maximum and minimum, which are used in nearly all cases for those definitions, other triangular norms could be used as well. Several definitions have been provided in the past, and by the so-called *representation theorem* of triangular norms there is a formal way to generate new triangular norms by the scheme of an already known one.

So far, triangular norms did not find much resonance in the pattern recognition community. This is an unlucky situation, since, at least in a formal sense, triangular norms allow for the formal extension of standard image processing operations and for the definition of new ones as well. From an image processing laymans point of view, triangular norms could be used whereever maximum or minimum formally appears in a definition. In a more mathematical sense, those norms serve as fusion operations, for which the standard operations maximum or minimum are just a very special choice. However, while all triangular norms have to fulfill the same set of mathematical requirements in order to be one, each of those norms is an individual one as well.

This paper explores the case of a very special triangular norm, which was proposed by Dubois and Prade in 1980 [1]. The interesting point is that this norm does not fit into the representational framework for generating new norms by means of generator functions. It is an outstanding triangular norm. This will be solicited for image processing as well. This viewpoint on fuzzy image processing was also followed in [7], where the exploitation of either fuzzy concept is encouraged, stimulated just by the practical usefulness of the derived image processing operations. However, some reasoning about the effects of the proposed operations are quite necessary as well, and this will be done in this paper, too.

In the following, in section 2 the definition of the Dubois and Prade triangular norm (and co-norm) is recalled. This procedure also serves as a model for reasoning about the effect of this norm on data, especially when it is used in image processing. The model is explained and demonstrated on some examples in section 2, too. Then, morphological operations based on the Dubois and Prade norm are introduced in section 3. Finally, in section 4, the application to the background removal of bankchecks is shown.

2. The Dubois and Prade fuzzy norm

2.1. Definition

The definition for a fuzzy norm were given by Dubois and Prade in 1980 [1]. Be a and b two real values from $[0, 1]$. Then, the triangular norm $T(a, b)$ (DPT) and co-norm (or S-norm) $S(a, b)$ (DPS) are given by the following formulas ($\alpha \in (0, 1)$):

$$T(a, b) = \frac{ab}{\max[a, b, \alpha]} \tag{1}$$

$$S(a,b) = 1 - \frac{(1-a)(1-b)}{\max[(1-a),(1-b),(1-\alpha)]} \quad (2)$$

2.2. Computational procedure

For computing the norms of more than two values, the law of associativity, which has to be fulfilled by each fuzzy norm, can be used in an iterative manner. Hence, for three values, the computation could be proceed as follows:

$$T(a,b,c) = T(T(a,b),c) \quad (3)$$

In order to get a more efficient procedure for computation of the DPT and DPS, the following simplifications can be made:

$$\begin{aligned} T(a,b,c) &= \frac{T(a,b)\cdot c}{\max[T(a,b),c,\alpha]} \\ &= \frac{\frac{ab}{\max[a,b,\alpha]}\cdot c}{\max[\frac{ab}{\max[a,b,\alpha]},c,\alpha]} \\ &= \frac{abc}{\max[a,b,\alpha]\cdot\max[\frac{ab}{\max[a,b,\alpha]},c,\alpha]} \\ &= \frac{abc}{\max[ab, c\cdot\max[a,b,\alpha], \alpha\cdot\max[a,b,\alpha]]} \\ &= \frac{abc}{\max[ab,ac,bc,\alpha a,\alpha b,\alpha c,\alpha^2]} \\ &= \frac{1}{\max[\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{\alpha}{ab},\frac{\alpha}{ac},\frac{\alpha}{bc},\frac{\alpha^2}{abc}]} \end{aligned}$$

We may finish our transformation at that point or consider the following relation:

$$\frac{1}{max[\frac{1}{x}]} = \min[x] \quad (4)$$

By applying this realation, the T-norm can be converted in a more “user-friendly” expression.

$$T(a,b,c) = \min[a,b,c,\frac{ab}{\alpha},\frac{ac}{\alpha},\frac{bc}{\alpha},\frac{abc}{\alpha^2}] \quad (5)$$

In the same manner the S-Norm for three elements can be derived and given as follow.

$$\begin{aligned} S(a,b,c) &= 1-\min[(1-a),(1-b),(1-c),\ldots \\ &\ldots \frac{(1-a)(1-b)}{(1-\alpha)},\frac{(1-a)(1-c)}{(1-\alpha)},\ldots \\ &\ldots \frac{(1-b)(1-c)}{(1-\alpha)},\frac{(1-a)(1-b)(1-c)}{(1-\alpha)^2}] \end{aligned}$$

Both equations do not consider any sorting of the elements a,b,c. Doing so, a reduction of the equations for T- or S-norm can be done by leaving out each element, which is greater or smaller resp. and has no effect in determining the minimum or maximum respectively. Lets assume the elements a,b,c are sorted in an incresing order, i.e. $a \le b \le c$, than the expression for $T(a,b,c)$ in equation 5 can be simplified to:

$$T(a,b,c) = \min[a,\frac{ab}{\alpha},\frac{abc}{\alpha^2}] \quad (6)$$

For the S-norm the elements a,b,c has to be sorted in an decreasing order to reduce equation 5.

$$\begin{aligned} S(a,b,c) &= 1-\min[(1-a),\frac{(1-a)(1-b)}{(1-\alpha)},\ldots \\ &\ldots \frac{(1-a)(1-b)(1-c)}{(1-\alpha)^2}] \end{aligned}$$

Doing the same for n values $a_1,a_2,\ldots,a_n$, the following general and closed expressions for the DPT and DPS can be derived:

$$T(\{a_n\}) = \begin{cases} a_1 & : a_2 > \alpha \\ \frac{1}{\alpha^{i-1}}\prod_{k=1}^{i} a_k & : a_i < \alpha \le a_{i+1} \end{cases} \quad (7)$$

(with $a_{n+1} = 1$) for the T-norm and

$$S(\{a_n\}) = \begin{cases} a_1 & : a_2 < \alpha \\ 1-\frac{1}{(1-\alpha)^{i-1}}\prod_{k=1}^{i} \bar{a}_k & : a_i > \alpha \ge a_{i+1} \end{cases}$$

(with $a_{n+1} = 0$ and $\bar{a}_k = 1 - a_k$) for the S-norm of Dubois and Prade.

2.3. Model

The result in equation 7 gives a means for reasoning about the effect of Dubois and Prade norm. This is based on the value of α, the only free parameter of the norm. In the following, only the T-norm is considered in more detail. The discussion will hold for the S-norm of Dubois and Prade in a similar manner.

In general, α allows for “tuning” the norm between two other norms. When α approaches 0, the expression for the DPT in eq. 1 will be a division of ab by the maximum of a,b and α, which will obviously not be α. Hence, ab is divided by the maximum of a and b, and the result will be the minimum of a and b. The same holds for n values: for α going to 0, the DPT will approach the mimimum of the values, i.e. their Archimedian norm. For α approaching 1, the norm in eq. 1 will be the product of a and b, since $\max[a,b,\alpha]$ clearly will choose α. This is just the algebraic norm of a and b, and this holds in the general case, too. We claim: The parameter α tunes the Dubois and Prade T-norm between the standard Archimedian norm and the algebraic norm.

For a more comprehensive understanding of the role of α, consider fig. 1. Assume again that the values a_1 to a_n

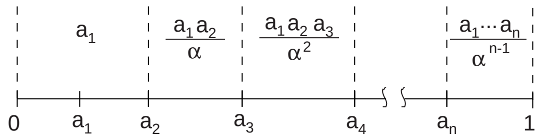


Figure 1. The computed value of the Dubois and Prade norm depends on the intervall of α.

are sorted in increasing order. Then, the value α will be in exactly one of the marked intervalls. If α is below the value of a_2, the value a_1 will be computed; when α lies between a_2 and a_3, the value $a_1 a_2/\alpha$ will be computed a.s.f. (note that the norm is continuously at the intervall borders). Since e.g. in the second case $\alpha \geq a_2$, the expression $a_1 a_2/\alpha$ is smaller than a_1. In general, the value of the Dubois and Prade T-norm is always smaller or equal to the minimum of the data values, from which the norm is computed.The higher α, the smaller the norm.

This gives some implications for the effect of the DPT on data. Assume e.g. the following cases:

- The values are equally distributed numbers from $[0,1]$. Then, α will in most of the cases lie within the same intervall, hence only similar values are computed. The noise will be reduced.

- The values are following a gradient, i.e. they are of the form $a_1 + k \cdot (a_n - a_1)$. Then, if a_1 goes from 0 to 1, the norm values will follow are more steep gradient.

- If all values are very small besides of only a few, the norm will be an even more smaller value, since α will fall into the intervall between the small and the large values.

- If all values are nearly equal, the T-norm values will also be nearly equal.

This are only some examples, how the model of fig. 1 can be used to undertsand the effects of norming data values by the Dubois and Prade fuzzy norm. A similar model exists for the S-norm, with the data values ranked in decreasing order. This will be omitted here.

3. Dubois and Prade fuzzy morphology

For using the Dubois and Prade fuzzy norm, morphological operations are derived from it. At each pixel x in an grayvalue image I, a fixed neighborhood is selected (e.g. the 4- or 8-neighborhood) by a structuring element. Then, from the grayvalues of the pixel and its neighbouring pixels, the DPT or DPS are computed. Note that the values have to be mapped from $[0, g_{max}]$ to $[0,1]$ before, and mapped back after the norm computation.

Such operations can be formally qualified as fuzzy morphology operations [5], since they give replacements for the infimum and supremum of a set of data, and basic definitions of mathematical morphology are derived from the requirement for an operation to commute with the supremum or infimum [3]. This can also be justified from the qualitative effects given in the last section (reduction of noise, broadening of dark areas for the Dubois and Prade T-norm, steepening of gradients). Since fuzzy morphology is an extension of grayvalue morphology, it is reasonable to use either freedom of choice (including the choice of a fuzzy infimum for a fuzzy erosion) to define such operations.

Fig. 2 shows a three times repeated horizontal gradient with a repeated and superimposed short vertical gradient. The effect of the Dubois and Prade fuzzy erosion in a 4-neighborhood can be seen in fig. 2, the effect of the corresponding Dubois and Prade dilation in fig. 2. Obviously, the horizontal gradients are more steep now. But also this demonstrates, how the Dubois and Prade morphological operations are able to “blend out” small background structures within an image, according to a shift in the value of α.

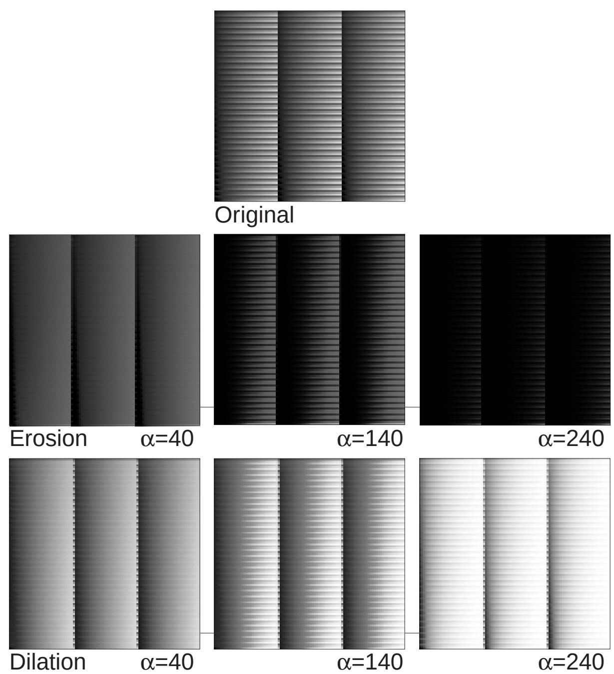


Figure 2. Dubois and Prade erosion and dilation of image with various values of α.

Finally, fig. 3 demonstrates noise removal by Dubois and Prade closing (i.e. first dilate, then erode with the same structuring element). The underlying fuzziness of the operation gives more soft object borders in the result image.

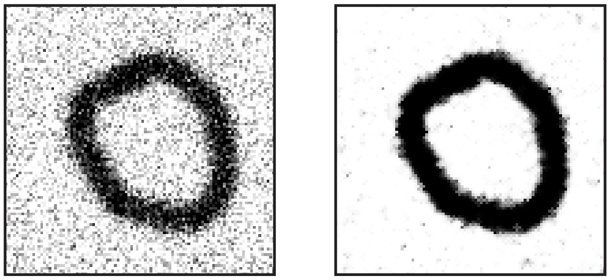
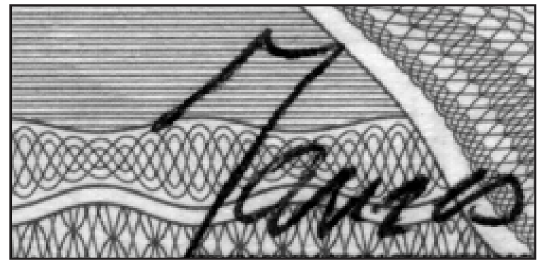


Figure 3. Noise removal by Dubois and Prade fuzzy opening.

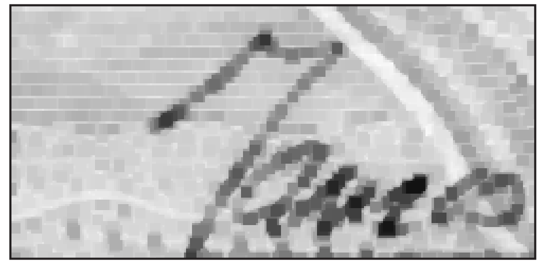
4. Application

The reason for studying the Dubois and Prade norm was within a more general work on background removal and handwriting analysis on bankcheck images. The texturing of the background of e.g. EC checks or traveller checks is a great problem for the extraction of the handwriting on the bankcheck, but has to be performed before any further processing can be started. In [2], a framework was presented, which is able to derive general foreground-background separating procedures on textured images by using genetic programming of image processing operations. However, this framework is as good as the base of image processing operations provided to the framework. A set of operations capable of background removal has to be selected.

Figure 4 shows the result of the application of standard morphology and Dubois and Prade morphology to such a bankcheck image. The image (a) was opened with standard grayvalue morphology and dilated afterwards, to obtain the image (b) (using a 3×3 structuring element), and it was Dubois and Prade opened with $\alpha = 0.8$ and dilated with $\alpha = 0.5$, using a 4-neighbor structuring element to obtain image (c). Despite of the interactive setting of reasonable values for α (which may not be suitable for other bankcheck images), the image (c) clearly shows the ability of Dubois and Prade morphology to completely remove the background while preserving a good quality of the handwriting. This is due to the effect of removing short gradients, which was demonstrated above. Such operations are a valuable addition to the set of basic operations needed for adaptive textured image processing according to the approach presented in [2].



(a)



(b)

(c)

Figure 4. Application of the standard grayvalue (b) and the Dubois and Prade fuzzy morphology (c) to the background removal of bankchecks.

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