A new approach to Fuzzy Morphology based on Fuzzy Integral and its application in Image Processing*

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Abstract

Based on similarities between fuzzy set theory and mathematical morphology Grabisch proposed a fuzzy morphology based on the Sugeno fuzzy integral [2]. This paper shows how these ideas can be applied to practical problems. Fast implementable definitions for erosion and dilation based on the fuzzy integral are given, their properties are discussed and some application examples are presented.

Keywords: mathematical morphology, fuzzy, fuzzy integral, fuzzy morphology, image processing

1. Introduction

A central problem for the design of image processing algorithms is the inclusion of the human intention about what a specific algorithm should do. As known from soft-computing this task is best performed by using fuzzy logic. Fuzzy logic allows data driven examination of human knowledge represented as a set of fuzzy rules or definition of fuzzy sets. The advantage of fuzzy logic in image processing results from two reasons, the possibility to overcome the crisp nature of pattern descriptions as they can be used for algorithms and the inclusion of human intentions in the process of algorithm goal formulation.

Mathematical morphology [4] has been succesfully applied to many problems of image processing, e.g. segmentation , thinning or object recognition . Most of the operations and filters defined in morphological operations also introduce a crisp element, the structuring element or mask. This depends on the nature of the image function used. A common approach to use fuzzy set theory is to define a fuzzy image function and generalize the conventional morphology to apply for this new kind of image [3]. Formally this approach is only a slight modification of the conventional 3D morphology. The region of the image to which the structuring element is applied is not interpreted as a fuzzy set, it is rather considered as a conventional set of image picture elements (pixels).

Once started considering a subset of pixels of an image as a fuzzy set it is straightforward to use fuzzy integral, as introduced in [5], for the evaluation of a morphological operation. There are formal similarities between fuzzy set theory and mathematical morphology that lead [2] to the proposal of a new approach to fuzzy morphology based on fuzzy integral.

This paper shows how these ideas can be applied to practical problems. Fast implementable definitions for erosion and dilation based on the fuzzy integral are given, their properties are discussed and some examples for the application are presented. It is organized as follows: Several conventional definitions of mathematical morphology will be recalled in section 2. Section 2 also introduces the definiton of dilation and erosion based on fuzzy integral and details an implementable reformulation of this definition. In section 3 some new properties of this definition are explored using a handsome model for the operations involved, followed by a sample application that outperforms conventional morphology in section 4. Section 5 gives a short discussion on the subject how the masks of the proposed fuzzy morphology really represent ”fuzzyness” and how the design of a fuzzy mask could be based on human intentions. Finally, section 6 gives a short summary of the work presented in this paper.

2. Definitions of mathematical morphology

2.1. Conventional Morphology

Mathematical morphology treats with structure describing image operations. Most of the operations of mathematical morphology are based on local operators (e.g. filters) which separate specific image regions due to their structural

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diversity. For many applications the use of a morphological operation results in the enhancement, filtering, extraction or deletion of an image constituant. In general, mathematical morphology never marks or classifies an image constituant because it is based on local operators. It gives rather a base for a following final image evaluation.

The major part of morphological operations can be defined as a combination of two basic operations, dilation and erosion, and set operations like difference, sum, maximum or minimum of two images.

The definition of these operations mainly depends on the definition of the image function I(x). Here x represents an element of the definition area of the image function (in most cases a compact subset of $\mathbb{N} \times \mathbb{N}$), also called picture element (pixel). The domain of I(x) describes the nature of the images. Mathematical morphology preserves the nature of the image.

Morphological operations also make use of a structuring element M, which can be either a set or a function that correspond to a neighbourhood-function related to the image function (i.e. the assignment of neighbours to a pixel). Both, image function and structuring element, constitute a specific kind of morphological operations.

Further operators can be constructed by sequencing these basic operations. Dilation and erosion are like the axioms of a formal theory, so the mathematical properties of a morphological operation are guaranteed if the fundamental properties of the basic operations are verified.

In general, a dilation operator D is every operator that commutes with the maximum operation, an erosion E is every operator that commutes with the minimum operation.

If I(x) acts like $\mathbb{N} \times \mathbb{N} \rightarrow \{0,1\}$, I is called a binary image, the associated morphology is called binary morphology. There is a homomorphism between the image function I and the set of all pixels with image function value 1 (the “foreground pixels”). The structuring element M(x) is a function that assigns a subset of $\mathbb{N}\times\mathbb{N}$ to every pixel of the image function [1]. Then dilation and erosion are sets defined as:

$$D_M \circ I = \bigcup_{\{x|I(x)=1\}} M^T(x)$$

$$E_M \circ I = \bigcap_{\{x|I(x)=1\}} M(x)$$

Dilation is an increasing transformation, i.e. $(D_M \circ I) \supseteq I$, while erosion is a decreasing transformation.

If I(x) is a function $\mathbb{N} \times \mathbb{N} \rightarrow$ G, where G is a finite subset of $\mathbb{N}$ (mostly the values 0,1,2,...,255 representing greyvalues), it is called greyscale-morphology. Here the problem is to define a morphology which is consistent to binary morphology. That means a pseudo-binary-image, containing only two greyvalues (e.g. 0 and 255), has to be treated like a binary image using binary morphology. The solution is to use a mask-like structuring element as a set of offsets from a fixed image position. So M is a set of offsets i and the addition of pixels and offsets is defined due to the image lattice. This results in the definiton for greyscale-morphology:

$$D_M \circ I = \max_{i \in M}(I(x-i))$$

for dilation and

$$E_M \circ I = \min_{i \in M}(I(x+i))$$

for erosion.

Another approach for greyscale images is the 3D-morphology that extends the concept of the structuring element M from a set of offsets to a set of assignments M(i) of offsets i into $\mathbb{N}$. The definitions for dilation and erosion are as follows:

$$D_M \circ I(x) = \max_{i \in M}(I(x) + M(i))$$

$$E_M \circ I(x) = \min_{i \in M}(I(x) - M(i))$$

From dilation D and erosion E, independent of the type of image function and structuring element used, the following operations can be defined:

Opening:	$O_M \circ I = D_M \circ (E_M \circ I)$
Closing:	$C_M \circ I = E_M \circ (D_M \circ I)$
Morphological gradient:	$\rho_M^m \circ I = (D_M \circ I) - (E_M \circ I)$
External gradient:	$\rho_M^e \circ I = (D_M \circ I) - I$
Inner gradient:	$\rho_M^i \circ I = I - (E_M \circ I)$
White tophat:	$T_M^w \circ I = I - (O_M \circ I)$
Black tophat:	$T_M^b \circ I = (C_M \circ I) - I$

Further operations can be defined based on the use of a second structuring element. Details about various morphological operators can be found in [4].

2.2. Definition of Fuzzy Morphology

The definition of fuzzy morphology used here is based on the proposal of [2]. He showed that the quasi-Sugeno fuzzy integral defined with respect to a proper measure can form an algebraic dilation or erosion. All rank filters can be considered as special cases of such fuzzy integrals.

In our paper morphological operations are used which are derived from the standard Sugeno fuzzy integral (see e.g. [5], [6], [1]). To calculate such an integral a λ–fuzzy measure $w(A)$ is defined which gives every set $A \subseteq X$ a definite non-negative weight w. Here X is a finite discrete set

[1] This assignment is made by considering the structuring element as a mask (as used e.g. for a convolution operation) with an origin. The mask is put with its origin on a pixel position. The subset assigned to that pixel by the structuring element is the set of all pixels covered by the mask. The transpose M^T is the symmetric of M with respect to its origin.

of pixels x_i which may be given by the action of a structuring element M(x) onto a single pixel. According to [5] the λ–fuzzy measure $w(A)$ fulfills the axioms:

$$w(0) = 0, \quad w(X) = 1 \tag{1}$$
$$w(A) \leq w(B) \quad \text{if} \quad A \subset B \tag{2}$$
$$w(A \cup B) = w(A) + w(B) + \lambda w(A) w(B) \tag{3}$$
$$\text{for some} \quad \lambda > -1 \quad \text{and} \quad A \cap B = \emptyset$$

The value of λ is determined by the first axiom. Given the weights w_i of the Pixels x_i, a $\lambda > -1$ can be determined, such that

$$w(X) = \frac{1}{\lambda}\Big[\prod_{i=1}^{n}(1+\lambda w_i) - 1\Big] = 1, \quad w_i = w(\{x_i\}). \tag{4}$$

Axiom (3) defines the "λ–fuzzy sum" for two weights assigned to disjunct sets. It is symmetric and associative, so a unique weight for every set $A \subseteq X$ can be calculated. Consider a vector $\vec{g} = (g_1, g_2, \cdots, g_n)$ of greyvalues $g_i = I(x_i)$ which results from the action of the mask M(x) onto I. The Sugeno fuzzy integral of the g_i with respect to the λ–fuzzy measure w() (for finite discrete sets) is than given by

$$I(g_i, w()) = \max_{i=1..n}(\min(g_i, w(A_i)) \tag{5}$$
$$\text{with} \quad A_i = \{x_j \mid g_j \geq g_i\}$$

If a fixed weight w_i is assigned to every mask point of the mask M, thus defining the λ–fuzzy measure w, this delivers a algebraic dilation for the greyvalues $\{g_i\}$ as was shown in [2]. The corresponding erosion is given by the dual fuzzy integral with respect to the dual λ–fuzzy measure:

$$I^*(g_i, w^*()) = \min_{i=1..n}(\max(g_i, 1 - w(A_i^*)) \tag{6}$$
$$\text{with} \quad A_i^* = \{x_j \mid g_j \leq g_i\}$$

2.3. Implementation

In our experiments a 3x3 square M with its origin in the middle was used as the structuring element. To every maskpoint a fixed weight w_i is assigned. Now λ can be calculated from the weights by solving equation (4). Because of that is an algebraic equation of order $n = 8$, this has to be done numerically. As shown in [6] there exists a unique solution in $(-1, \infty)$. Now the mask M can be applied to every pixel of the image. The computation of the fuzzy integral requires sorting of the greyvalues in the mask. When the greyvalues are sorted in decreasing order, the $w(A_i)$ can recursively be calculated from the $w_i = w(\{x_i\})$ and be compared with the g_i. So the algorithm is as follows:

1. Sort the greyvalues[2] such, that $g_1 \geq g_2 \geq \cdots \geq g_n$.
2. Start with $w(A_1) = w_1$ and $I_1 = \min(g_1, w(A_1))$.
3. For $i \leq n$ recursively calculate:
   $w(A_i) = w(A_{i-1}) + w_i + \lambda w_i w(A_{i-1})$ and
   $I_i = \max(I_{i-1}, \min(g_i, w(A_i)))$
   The value I_n gives the fuzzy integral.

[2] The corresponding weights get a new order, too.

Similarly the dual fuzzy integral can be computed from the greyvalues sorted in increasing order. In the applications it can be helpful to create a lookup table for the weights $w(A_i)$ of all possible sets A_i of pixels in the mask. Then, the dual fuzzy integral can also be computed from decreasing order, thus speeding up the opening and closing operations.

3. Properties of the fuzzy integral

As shown in [6] there exists a unique $\lambda > -1$, which solves equation (4). Its sign depends on the sum of the weights. If the sum of the weights w_i is greater than 1, λ is negative, if it is lower than 1, λ is positive. In the case of $\sum w_i = 1$ the measure is a (additive) probability measure with $\lambda = 0$.

Since all w_i lie in (0,1) the factors $(1 + \lambda w_i)$ in equation (4) vary only weakly. Therefore it is a reasonable approximation to replace their geometric average by the arithmetic average. This leads to:

$$\prod_{i=1}^{n}(1+\lambda w_i) \approx \Big[\frac{1}{n}\sum_{i=1}^{n}(1+\lambda w_i)\Big]^n = (1+\lambda\bar{g})^n$$
$$\text{with} \quad \bar{g} = \frac{1}{n}\sum_{i=1}^{n} w_i$$

That means that the value of λ only shows a weak dependence on the distribution of the w_i. λ is determined rather by the sum of the weights. Figure 1 shows the dependence of λ from the sum of weights in the case of $n = 9$. In comparison to the case of equal weights two curves for a distribution of two different weights varying by a factor of 10 and 100 are shown. As predicted the three curves differ only very little.

The fuzzy integral is a kind of average. Its value is determined by the crossover[3] of the decreasing curve of the ordered greyvalues in the mask with the increasing curve of the λ–fuzzy sum of weights. Figure 2 shows two examples. If the upper greyvalues have high weights the crossover shifts to the left delivering high values for the integral. On the other side, if low weights are assigned to the upper greyvalues, the crossover shifts to the right giving lower values for the integral.

It is worth to consider two interesting limiting cases which deliver upper and lower bounds for the value of the fuzzy integral. As can be easily shown the λ–fuzzy sum of two weights is always greater than the minimum of the

[3] If they have more than one point in common, the minimum has to be taken.

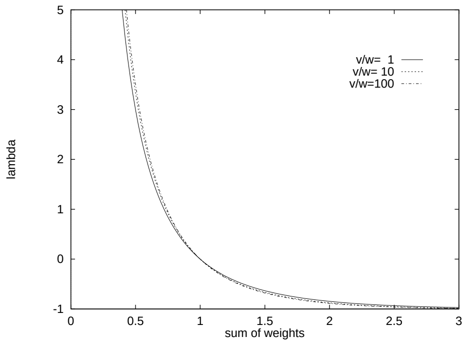

Figure 1. Dependency of λ on $\sum w_i$ for $\vec{w} = (v, v, v, v, v, w, w, w, w)$ with three different values: $v = w$ (equal weigths), $v = 10w$, $v = 100w$

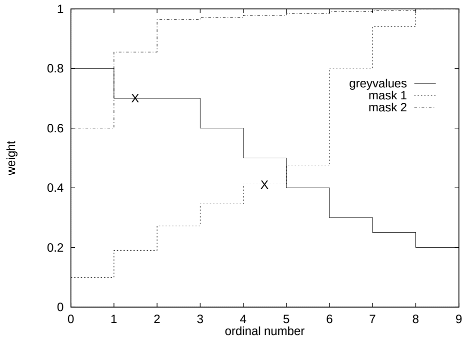

Figure 2. Evaluation of fuzzy integral for $\vec{w}$ = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.6, 0.6, 0.6) (mask 1) and $\vec{w}$ = (0.6, 0.6, 0.6, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1) (mask 2). The values of the corresponding fuzzy integrals are marked with crosses.

two weights. Therefore, if there are one or more weights w_i which tend to 1 whereas all other weights are small, then the fuzzy integral just gives the maximum greyvalue of the mask points with these weights. In this case, the standard greyscale morphology is reproduced. If all weights are much smaller than 1 the λ–fuzzy sum of the weights will increase only at the very end when reaching the low greyvalues. In this case the value for the fuzzy integral approaches the minimum of all the greyvalues in the mask. The two cases are shown in figure 3.

4. Application

The problem to solve was to detect infected regions of a leaf (see figure 4a). In the image of the leaf the infected regions occur as slightly brightened regions. From a local point of view the pixels with slightly higher greyvalues are not connected, so it was useful to apply an opening operation. The first erosion darkens isolated bright pixels, the

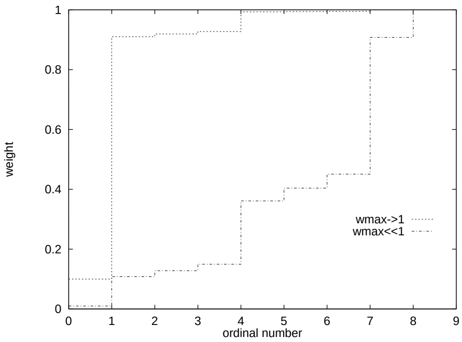

Figure 3. λ–fuzzy sum of the weights for $w_{max} \rightarrow 1$: $\vec{w}$ = (0.1, 0.9, 0.1, 0.1, 0.9, 0.1, 0.1, 0.9, 0.1) and $w_{max} << 1$: $\vec{w}$ = (0.01, 0.09, 0.01, 0.01, 0.09, 0.01, 0.01, 0.09, 0.01)

following dilation connects the remaining regions of bright pixels (i.e. brightens the gaps). After that a two-threshold binarization [4] is applied to separate leaf background and infected regions. A morphological gradient operator and the addition of this result and the original image are used for marking the infected regions on the original image. In figures 4b-d the gradient marks the regions found by a white border surrounding them. The result for using greyscale morphology with a structuring element of shape

o	x	o
x	x	x
o	x	o

is shown in figure 4b. Using 3D morphology with a mask with values

3	8	3
8	8	8
3	8	3

results in the marked white regions shown in figure 4c. The regions in both results are badly connected and scattered over the infected areas.

Finally the fuzzy morphology using the structuring element

[4] This operation acts by two given thresholds, a weak one and a strong one. This operation selects region of the image with greyvalues larger than the strong threshold, but surrounded by regions of the image with greyvalues larger than the weak treshold. This is done by reconstruction of the set of all pixels brighter than the strong threshold by the set of pixels brighter than the weak treshold. The reconstruction operation is implemented by a flooding algorithm. In the case of our application the weak and strong thresholds were 80 and 150 resp.

0.3	0.8	0.3
0.8	0.8	0.8
0.3	0.8	0.3

was applied for the same operation. The satisfying result, which can be seen in figure 4d, comes from the fact that the crosslike shape of the structuring element is a little bit fuzzyfied. So, the operation achieves a compromise between the removement of isolated bright pixels and the connection of scattered regions of bright pixels.

5. Discussion

Fuzzy morphology, as defined in this paper, has the same advantages as other kinds of morphology. Also there is a fast implementation, so the computational overhead can be kept small. The advantage over other kinds of morphology is just what its name says, its fuzzyness. Normally a morphological operator tends to work as a crisp filter, recognizing only the exact match of an image pattern and the pattern described by the structuring element. Fuzzy morphology allows a tuning of the requested correctness of a match between patterns.

Figure 5a shows a binarized herringbone pattern texture which contains two direction primitives. Performing a binary dilation of size 3 with a structuring element (mask) of the pattern

x	o	o
o	x	o
o	o	x

results in the image shown in figure 5b. The direction discrimination of this operation can be clearly seen. The result of applying the same operation (dilation of size 3) with a mask of similar shape

x	0.1	0.1
0.1	x	0.1
0.1	0.1	x

and x being a real from the set $\{0.9, 0.7, 0.5, 0.3, 0.1\}$ gives the results shown in figures 5c-g resp.

This way the preference of the filter for the 45 degree direction can be tuned by modifying the x parameter in the structuring element. This allows the perpendicular 135 degree direction to be also partly detected by this filter.

The example shows, how fuzzy morphology introduces fuzzyness in morphological operations. By proper choosing mask values the designer of image processing algorithms can take care for uncertainties in her goal and for noise in images. Also it is possible to parametrize morphological operations and to make them a subject of adaptation.

6. Summary

A new approach to fuzzy morphology has been introduced. This approach is based on an idea given by [2] which uses the Sugeno fuzzy integral for the definition of a new kind of mathematical morphology. It is extended to an implementable formulation of fuzzy morphology.

The structuring element is considered as a fuzzy set. By describing it as a mask the mask values are used as fuzzy densities and a measure for subsets of the mask can be defined. This measure is the λ–fuzzy measure as defined in fuzzy set theory.

Also the subset of pixels covered by the mask is considered as a fuzzy set. Hence, dilation and erosion can be defined by fuzzy integration of this fuzzy pixel set according to the fuzzy measure defined by the mask. The dilation is just the Sugeno fuzzy integral, the erosion is given by its dual.

The so defined operations have the same properties as the conventional morphological operations.

An application example (detection of infected regions of a leaf) demonstrates the ability of fuzzy morphology to be more sensitive for specific image substructures. Also, another advantage of the proposed approach was shown in section 5. By modifying mask weights it is possible to tune morphological operations to specific image processing tasks. This allows the adaptation of fuzzy masks, a possibility currently under study.

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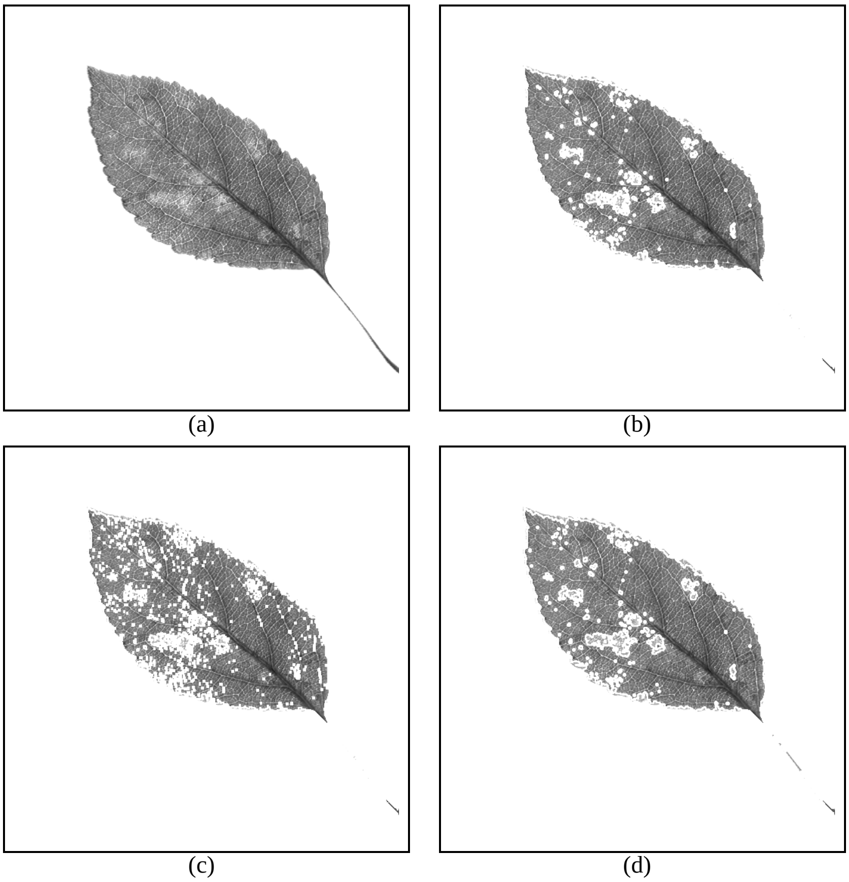
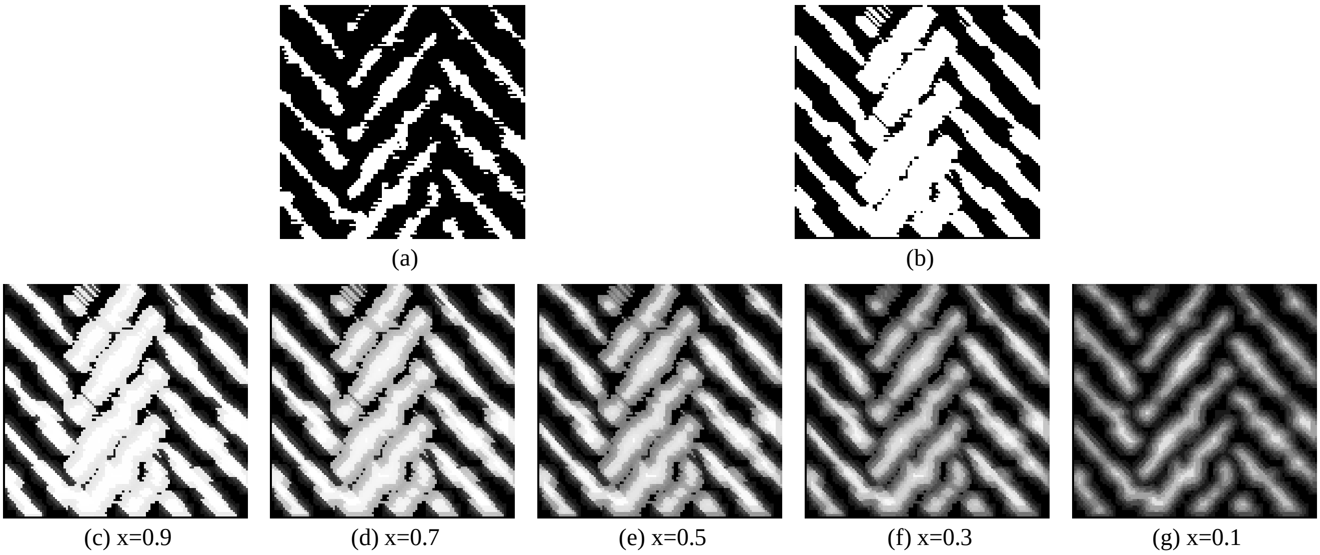


Figure 4. Application of fuzzy morphology: (a) Image of an infected leaf, (b) Opening using greyscale morphology, (c) Opening using 3D morphology, (d) Opening using the proposed fuzzy morphology



Figures 5c-g: Dilation of size 3 using fuzzy morphology for different values of x

Figure 5. (a) Binarized textured image, (b) Dilation of size 3 using binary morphology, (c)-(g) Dilation of size 3 using fuzzy morphology for different values of x