A Survey on Fuzzy Morphology[1]

M. Köppen*, K. Franke*, O. Unold**

**Fraunhofer IPK Berlin, Pascalstr, 8-9, 10587 Berlin, Germany*
e-mail: mario.koeppen@ipk.fhg.de, katrin.franke@ipk.fhg.de
***Wroclaw University of Technology, Wyb. Wyspianskiego 27, 50-370 Wroclaw, Poland*
e-mail: unold@ci.pwr.wroc.pl

Abstract—This paper considers various aspects of generalizing mathematical morphology towards a fuzzy discipline. Since mathematical morphology and fuzzy theory are both based on a set theory, there are many approaches for defining dilation and erosion operations of a fuzzy morphology. Among those approaches are: the alpha-Morphology; the triangular norm based approach of Bloch; the logical approach of Sinha and Dougherty and de Baets; and the fuzzy fusion based approaches.

Mathematical Morphology comprises an important toolset for analyzing spatial structures in images [12, 19, 20, 23]. For binary images, the definitions of the fundamental morphological operations—dilation and erosion—can be related to the set-theoretic Minkowski addition and subtraction. The extension of those operations to grayscale images is strongly related to ranking operations and, therefore, to the concept of ordered sets.

It has been considered for a long time how to extend mathematical morphology to the case of fuzzy sets (as was done in other image processing disciplines, e.g., see [18]). Although there was a simple idea to consider grayscale images as fuzzy versions of binary images, further works concentrated on the solicitation of a more reasonable and nontrivial concept of a fuzzy morphology [6, 8, 11, 13, 27]. Those works have culminated in the proposal of a new operation, using fuzzy structuring elements and basing on fuzzy level sets (referred to as alpha-morphology in the following).

Now, it becomes more and more obvious that the intermixing of two concepts, which are primarily based on set theory (fuzzy logic and mathematical morphology), leads to a multitude of different approaches. There were also proposals of more and more operations, which also fuzzify the standard morphological operations, but differ entirely from alpha-morphology.

This paper gives a survey on the various concepts of a fuzzy mathematical morphology; the most prominent fuzzy morphology, the alpha-morphology, is introduced. This approach is based on the level (or alpha-) sets of a fuzzy membership degree function. Since each level set is a classical set by itself, standard morphological operations can be applied to them. After doing so for each level, the dilated or eroded level sets can be reunified to a grayscale image. Bloch *et al.* provided a formula for simplifying this computation [6].

However, by investigating the use of triangular norms in this simplification, Bloch *et al.* discovered a further family of fuzzy morphologies [5, 6]. For the first time, such operations have a formal degree of freedom, by allowing for replacing the occurrence of a minimum operation within the analytical expression of the

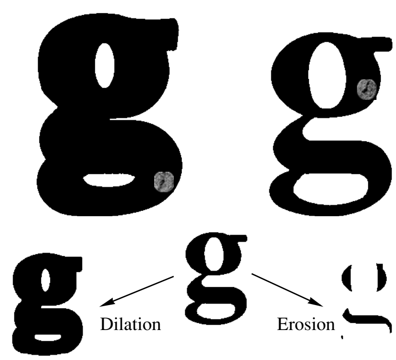

Fig. 1. Dilation and erosion of the letter “g” by a circular structuring element.

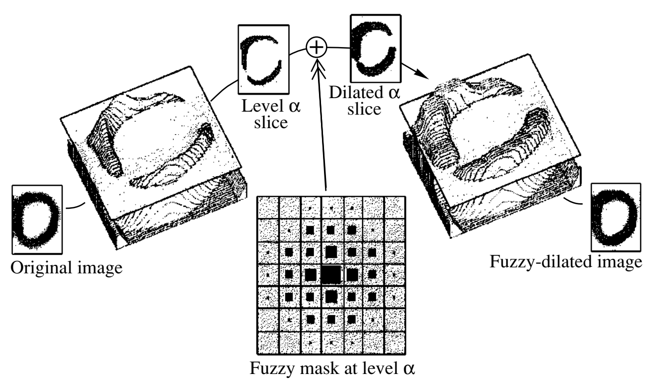

Fig. 2. α-morphology: the original grayscale image is sliced at level α, the slice image is dilated by the sliced mask, and the results of each slice level are recombined to the result image.

[1] This article was submitted by the authors in English.

Received October 25, 2000

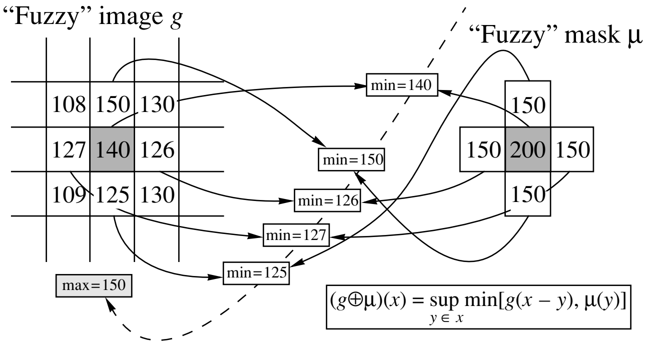

Fig. 3. Illustration for better understanding the Bloch formula: α-Morphology can be simplified by yielding the maxmin of image and mask positions, where image offsets correspond to positions of the *point-inverted* mask.

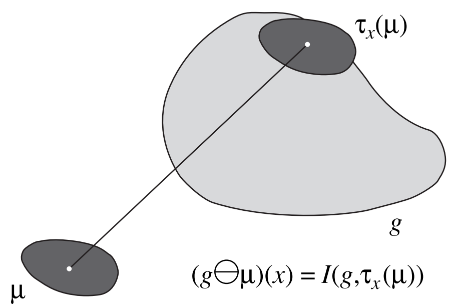

Fig. 4. General procedure of the Sinha and Dougherty approach to fuzzy erosion, which generalizes and fuzzifies the Minkowski subtraction.

operation with either formal expression of a triangular norm.

Also, Sinha and Dougherty [16, 21, 22] in their approach considered the interpretation of the inclusion operation involved in the definition of an erosion as a logical operation. Then, this logical operation was fuzzified by means of a so-called inclusion indicator (i.e., by generalizing an "if-then" [15]). Later on, the approach was formally refined by de Baets [1–4].

The development of several approaches to fuzzy morphology was accompanied by the proposal of mathematical requirements which had to be fulfilled by a new fuzzy morphology. The most important set of requirements was provided by Bloch [6], but this was far from being perfect. The most general requirements were given by Serra [19, 20], who basically introduced a concept of a supremum and possibility to define many operations from it (a very practical point of view). Further requirements ought to be reasonable only in the sense that they may help to qualify a new operation and judge a field of application. It has to be noted that the meaning of some requirements can be fuzzified as well.

Each approach to a fuzzy morphology gives a certain aspect of the underlying reasoning which has to be fuzzified. This could be the image data (grayscale images), the structuring element, or the set inclusion involved in the definition of Minkowski subtraction. We also propose the fuzzification of the pixel neighborhood, which leads to a further formal approach to fuzzy morphology using triangular norms and co-norms, i.e., based on general concepts of fuzzy information fusion [7, 9, 10, 24, 25, 28]. This is exemplified by the triangular norm provided by Dubois and Prade.

Also, the case of multivariate data (color morphology) is shortly considered. It comes out that fuzzy fusion concepts can be used to extend mathematical morphology to color data as well [14].

The detailed study of fuzzy morphologies can be considered as part of the efforts to achieve a more general viewpoint on image processing itself [17, 26].

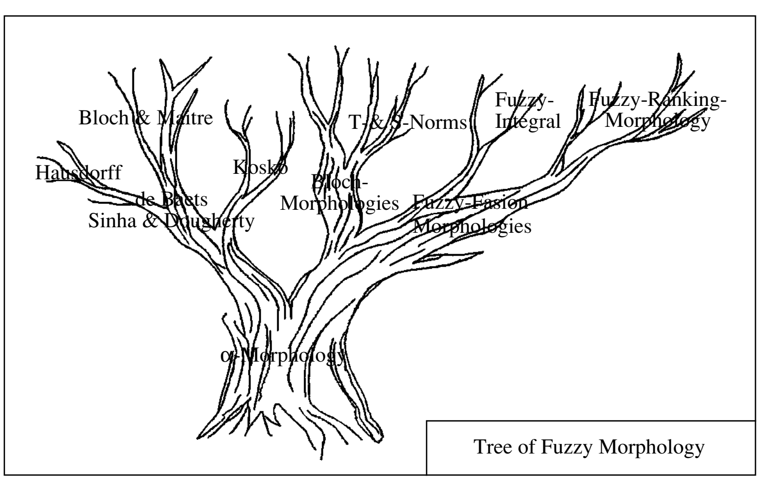

Fig. 5. The lineage of fuzzy morphologies, as they evolved from each other.

REFERENCES

1. de Baets, B., Kerre, E., and Gupta, M., The Fundamentals of Fuzzy Mathematical Morphology. Part 1: Basic Concepts, *Internat. J. Gen. Systems*, 1994, vol. 23, pp. 155–171.
2. de Baets, B., Kerre, E., and Gupta, M., The Fundamentals of Fuzzy Mathematical Morphology. Part 2: Idempotence, Convexity and Decomposition, *Internat. J. Gen. Systems*, 1995, vol. 23, pp. 307–332.
3. de Baets, B., *Generalized Idempotence in Fuzzy Mathematical Morphology*, in Fuzzy Techniques in Image Processing (Kerre, E. and Nachtegael, M., Eds.), Studies in Fuzziness and Soft Computing, Physica, 2000, pp. 58–75.
4. de Baets, B., *Fuzzy Morphology: A Logical Approach*, in Uncertainty Analysis in Engineering and Sciences: Fuzzy Logic, Statistics and Neural Network Approach (Ayyub, B. and Gupta, M., Eds.), Kluwer Academic Publishers, 1997, pp. 53–67.
5. Bloch, I. and Maître, H., Constructing a Fuzzy Mathematical Morphology. in *Proc. FUZZ-IEEE'93*, San Francisco, CA, 1993, pp. 1303–1308.
6. Bloch, I. and Maître, H., Fuzzy Mathematical Morphologies: A Comparative Study, *Pattern Recognit.*, 1995, vol. 28, pp. 1341–1387.
7. Dubois, D., Fargier, H., and Prade, H., *Beyond Min Aggregation in Multicriteria Decision: Ordered Weighted Min*, *Leximin*, in The Ordered Weighted Averaging Operators—Theory and Applications, (Yager, R. and Kacprzyk, J., Eds.), Kluwer Academics Publishers, 1997.
8. Gader, P., Fuzzy Spatial Relations Based on Fuzzy Morphology, in *Proc. FUZZ-IEEE'97*, Barcelona, 1997.
9. Grabisch, M., Fuzzy Integral in Multicriteria Decision Making, *Fuzzy Sets and Systems*, 1994, vol. 69, pp. 279–289.
10. Grabisch, M., Murofushi, T., and Sugeno, M., Fuzzy Measure of Fuzzy Events Defines by Fuzzy Integrals, *Fuzzy sets and systems*, 1991, vol. 50, pp. 293–313.
11. Grossert, S., Köppen, M., and Nikolay, B., A New Approach to Fuzzy Morphology Based on Fuzzy Integral and Its Application in Image Processing, in *Proc. ICPR'96*, Vienna, 1996, pp. 625–630.
12. Heijmans, H., *Morphological Image Operators*, Boston, MA: Academic, 1994.
13. Kaufmann, A. and Gupta, M.M., *Fuzzy Mathematical Models in Engineering and Management Science*, Amsterdam: North-Holland, 1988.
14. Köppen, M., Nowack, C., and Rösel, G., Paredo-Morphology for Color Image Processing, in *Proc. SCIA'99*, Kangerlussuaq, Greenland, 1999.
15. Kosko, B., *Neural Networks and Fuzzy Systems*, Prentice Hall, 1991.
16. Popov, A.T., Convexity Indicators Based on Fuzzy Morphology, *Pattern Recognit. Letters*, 1997, vol. 18, pp. 259–267.
17. Ritter, G.X. and Wilson, J.N., *Handbook of Computer Vision Algorithms in Image Algebra*, CRC, 1996.
18. Rosenfeld, A., Fuzzy Plane Geometry: Triangles, *Pattern Recognit. Letters*, 1994, vol. 15, pp. 1261–1264.
19. Serra, J., *Image Analysis and Mathematical Morphology*, London: Academic, 1982.
20. Serra, J., *Image Analysis and Mathematical Morphology: Theoretical Advances*, Academic, 1988.
21. Sinha, D. and Dougherty, E., Fuzzification of Set Inclusion, Theory and Application, *Fuzzy Sets and Systems*, 1993, vol. 55, pp. 15–42.
22. Sinha, D. and Dougherty, E., A General Axiomatic Theory of Intrinsically Fuzzy Mathematical Morphologies, *IEEE Transactions of Fuzzy systems*, 1995, vol. 3, pp. 389–403.
23. Soille, P., *Morphological Image Analysis: Principles and Applications*, Berlin: Springer, 1999.
24. Sugeno, M., *Theory of Fuzzy Integral and Its Applications*, PhD thesis, Tokyo Inst. of Technology, 1974.
25. Tahani, H. and Keller, J., Information Fusion in Computer Vision Using Fuzzy Integral, *IEEE Trans. on System, Man and Cybernetics*, 1990, vol. 20, pp. 733–741.
26. Watanabe, S., *Pattern Recognition: Human and Mechanical*, John Wiley and Sons, 1985.
27. Werman, M. and Peleg, S., Min-max Operators in Texture Analysis, *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 1985, vol. 7, pp. 730–733.
28. Yager, R., On Ordered Weighted Averaging Aggregation Operators in Multi-criteria Decision Making, *IEEE Trans. on SMC*, 1988, vol. 18, pp. 183–190.
29. Zaden, L.A., *Fuzzy Sets and Their Application to Cognitive Processes*, London: Academic, 1975.