Pareto-Morphology for Color Image Processing

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Abstract

This paper presents an approach to the generalization of grayscale morphology to color images. Attaining such a generalization is strongly related to the issues of multivariate ranking and to the Pareto sets of multiobjective optimization. Some ranking schemes for multivariate data are recalled. For color morphology, the most important underlying ranking scheme is reduced ordering (also referred to as total ordering). Also, there is the partial ordering, which gives the important class of Pareto-Morphologies. Since partial ordering by Pareto sets commutes with reduced ordering, a so-called Pareto-Morphology is defined as a generalized multivariate morphology, for which the results will not change, when its computations are restricted to the Pareto set of the (local) neighborhood of a pixel. By further applying the concept of fuzzy subsethood to color values, a Pareto-Morphology can be designed, which is not based on reduced ordering, hence providing a manner for native color treatment. The properties of this newly-proposed Fuzzy-Pareto-Morphology and examples of its application for the processing of color textile images are given.

1 Introduction

Mathematical morphology can be considered as a theoretical and practical means for analyzing spatial structures. It comprises a versatile toolset of techniques for image processing, whose usefulness has been proven for the processing of binary images and grayscale images as well. Operations of mathematical morphology are image-to-image transformations based on a structuring element, which acts like a probe sensitive for structural information. As a result of the operation, some image features might be enhanced, suppressed or preserved [10].

Basically there are two morphological operations, dilation and erosion, which are used for the definition of more complex morphological operations. Nowadays, definitions of dilation and erosion are fixed for the treatment of binary images and grayscale images. Other concepts includes the generalization of these basic definitions, considering aspects like higher dimensions or fuzzy logic (consider e.g. [2][6][9]).

However, requirements for a generalized dilation as an image-to-image operation, which employs a structuring elements, are still discussed. As suggested in [8], there should be three key ideas, based on which the dilation is defined[1]: an idea of ranking due to a sort order; an idea of a supremum due to this ranking; and the possibility of admitting an infinity of operands.

This paper deals with the definition of dilation (and erosion) within the context of color image processing. The fundamental lack of a “natural sort order” of multivariate data and the numerical differences due to the choice of different color spaces make it hard or even impossible to define something like a “color morphology.” But it could be expected to transfer a large number of grayscale morphological techniques to color images.

Very few past work dealt with such extensions. In [4], the issue was intensively discussed and a definition of a color morphology was presented, which will be generalized in this paper as belonging to a larger class of color morphologies, each of which is based on its own unique definition of the dilation operation. In [3], a color morphology for the processing of label images, i.e. images, wherein each pixel position is labeled by a color, indicating e.g. class membership, is proposed. This approach is very useful for the morphological treatment of e.g. classification and segregation results, but it lacks some consistency within its basic definitions and it assumes only a few colors to be present within the image.

In general, attaining a multivariate or color morphology is strongly related to the issues of multivariate ranking and to the Pareto sets of multiobjective optimization. The fundamental classification of ranking schemes for multivariate data was given in [1]. For color morphology, the most important underlying ranking scheme is reduced sorting (also referred to as total ordering). The approach of [4] belongs to this class. Also, [1] considers the partial ordering. This gives a new class of multivariate morphologies. Since partial ordering by Pareto sets commutes with reduced ordering based on monotonic scalar functions, a so-called Pareto-Morphology is defined as a generalized multivariate morphology, for which the results will not change, when its computations are restricted to the Pareto set of the (local) neighborhood of a pixel. As stated, multivariate morphologies

[1]Generally, when the definition of a dilation is fixed, the erosion is defined as the complementary operation.

based on reduced ordering partially belong to that class.

The core of this paper presents a new Pareto-Morphology for multivariate data (including color images), which is truely not based on reduced ordering, comprising a more "native" treatment of color images. It is based on the concept of fuzzy-subsethood, as introduced by [7], and the notion of maxmin/minmax-operations as possible extensions of the "one-dimensional" maximum operation, what was not seen by Barnett in his seminal paper as an approach to multivariate ranking [1].

This paper is organized as follows. Section 2 guides to the definition of the newly-presented Fuzzy-Pareto-Morphology (FPM). There, some requirements are given for generalized dilation operations, the concept of a Pareto set is recalled, Pareto-Morphology is defined and discussed, the definition of fuzzy subsethood is recalled and finally, the definition of the FPM is given. Then, in section 3, the FPM is discussed. Some of its properties are given, and its relation to the requirements for a generalized dilation is considered. Section 4 presents other operations, which could be defined on base of the fuzzy-subsethood concept as well. Section 5 presents examples of fault detection in color textiles to demonstrate the usefulness of the newly-presented approach. The paper ends with a short summary, an acknowledgment and the reference.

2 Fuzzy-Pareto-Morphology

2.1 Requirements for a generalized dilation

Be $p : D_p \subset \mathbf{Z}^2 \longrightarrow \{0, \ldots, p_{max}\}^3$ a two-dimensional color image function, with D_p its domain, usually a rectangular subset of the plain, and p_{max} the maximum intensity value for one color channel. The chosen color space should be fixed in the following. Be a and b structuring elements, e.g. defined as a set of offsets with $a, b \subset \mathbf{Z}^2$ and with respect to a central point. Thus, a structuring element defines a neighborhood M for each pixel of the image. Then, a generalized dilation $\oplus$ will assign a new color value p_{new} for each image position (x, y) by means of its defining set function $P : M \subset \mathbf{U} \rightarrow m \in \mathbf{U}$, with $\mathbf{U}$ a suitable superset of the neighborhood, as follows:

$$p_{new}(x,y) = P \circ \{p(k,l) | k = x+i, l = y+j, i, j \in a\}.$$

The following requirements are considered to be important for a generalized dilation operation $\oplus$ for color images:

1. The dilation should be vector-preserving, i.e. P should just select one color value out of the set M of color values within the neighborhood of a pixel. This is quite important for the processing of color images, due to the fact, that newly introduced colors within the result image might appear as artifacts or cluttering. This requirement could be relaxed by using the HSI color model and preventing only the introduction of new H components.
2. A generalized dilation should be an increasing operation, i.e.
$$p \oplus a \geq p,$$
where the meaning of $\geq$ is according to the key idea of sorting, as it was mentioned in the introduction. It could also be said, that a dilation commutes with the supremum.
3. When $\oplus_B$ assigns standard binary morphology, the generalized dilation should be compatible with this operation, i.e.
$$(p \oplus a) \oplus b = p \oplus (a \oplus_B b).$$
4. In the context of multivariate data, a generalized dilation should become a standard grayscale dilation, if the definition of P is restricted to the one-dimensional case.

2.2 Multivariate ranking

In order to fulfill requirements 1 and 2, the concept of ranking n values should be extended to the ranking of n vectors. In [1], ordering principles were classified into four categories: marginal ordering, reduced ordering, partial ordering and conditional ordering. Only marginal ordering, reduced ordering and partial ordering can be used in order to define the set function of a generalized dilation. An example for a generalized dilation based on marginal ordering is given by applying a one-dimensional set function P component-wise:

$$P \circ \{(p_x, p_y, p_z)\} = (P \circ \{p_x\}, P \circ \{p_y\}, P \circ \{p_z\}).$$

However, marginal ordering would produce new color values within the result image, thus for color images violating the strong requirement 1.

By reduced ordering (also called total ordering), a scalar parameter function is computed from the vector components of each color value within M. The ranking is performed according to the resulting scalar values. This could be e.g.

$$P \circ \{(p_x, p_y, p_z)\} = \underset{p \in M}{\operatorname{argmax}} \ (p_x + p_y + p_z),$$

with the scalar function $f(x,y,z) = x+y+z$. This approach was used in [4] for defining a generalized morphology for color images. Either function f could be used, e.g. $f(x,y,z) = \max(x,y,z)$ or $f(x,y,z) = xy - z$. It fulfills requirements 1, 3 and, according to the definition of f also requirement 4. Its relation to the requirement 2 will be discussed in the next subsection.

The essential point here is the intrinsic transformation of the color image to a single-channel image by taking the values of the scalar function at each image point. Then, the dilation could be applied to this channel image as well. Reduced ordering does not relate the color values within a structural neighborhood. It assigns the ranking value to each color in before-hand. Every result achieved from this approach could be achieved from a grayscale image (derived from the scalar values of f) as well. Color morphology based on reduced ordering merely becomes grayscale morphology.

The third ranking scheme, partial ordering, differs from marginal and reduced ordering by possibly selecting more than one value out of the set of the unsorted values. Partial ordering iteratively strips off subsets of the data set, until the remaining set is empty. A common choice for a set function P is the convex hull of the data set. In order to use partial ordering for generalizing dilation operations, an additional procedure has to be specified for selecting exactly one value out of the set of the stripped off subsets.

There is a comparable problem in the field of multiobjective optimization. Recent approaches here make use of the concept of a Pareto set [5]. If two vectors $\vec{a}$ and $\vec{b}$ are to be compared, it is said that $\vec{a}$ dominates $\vec{b}$, when each component of $\vec{a}$ is at least as large as the corresponding component of $\vec{b}$, and at least one component is larger:

$$\vec{a} >_D \vec{b} \longleftrightarrow \forall i(a_i \geq b_i) \wedge \exists k(a_k > b_k).$$

The subset of all vectors, which are not dominated by another vector, is the Pareto set (also Pareto front). We consider the subset operator **P**, which assigns the Pareto set to a set of vectors.

The Pareto set describes the possible solutions of a multiobjective optimization problem. According to the problem statement, every solution, which gives an element of the Pareto set, when its multiple criteria are computed, is optimal in this generalized sense.

2.3 Pareto-Morphology

The concept of partial ordering based on the Pareto sets allows for a reformulation of requirement 2, in order to have a better specification of what is meant by $\geq$. If P is a monotonic increasing scalar function used for a reduced ordering, it gives the same results if this reduced ordering is applied first and the Pareto set is computed then (the Pareto set of a onedimensional data set is simply its supremum), or if the reduced ordering is applied to the Pareto set first and the supremum is selected then. With other words, in this case, the Pareto operator **P** commutes with the reduced morphology.

Hence, the operator **P** comprises a natural extension for the basic idea of dilation operations to be operations which commute with the supremum to the multivariate case. Requirement 2 is now formulated as follows:

The result of the dilation must not be dominated by its original value.

$$\neg \exists p : \quad p >_D p \oplus a.$$

A generalized dilation is said to fulfill the Pareto property, if its set function P gives the same result for a neighborhood M, if restricted to $\mathbf{P}(M)$.

Every morphology derived from a generalized dilation with the Pareto property is referred to as Pareto-Morphology.

If the parametric function f is non monotonic, the reduced ordering may violate the Pareto-property. As an example, consider $f(x,y) = \sin\frac{\pi}{2}x + sin\frac{\pi}{2}y$ and its values $f(1,1) = 2$, $f(2,2) = 0$ and $f(1,3) = 0$. While the Pareto set is given with $\{(2,2),(1,3)\}$, the supremum point select by $f(x,y)$ is (1,1).

At this point, the important question comes up, whether there exists a Pareto-Morphology, which is not based on reduced ordering. The next two subsection will give the answer to this intriguing question.

2.4 Fuzzy subsethood

Generally, a fuzzy set is given by the membership degrees of its elements

$$M = (\mu_1, \mu_2, \ldots, \mu_n)$$

According to the fuzzy set approach of Kosko [7], fuzzy sets could be considered as points in the n-dimensional unit square (or unit cube) by using the membership degrees as coordinates. If the parallels to the coordinate axis are drawn, a hyperrectangle is segregated. Hence, each fuzzy set corresponds to a hyperrectangle in the unit square. Figure 1 shows two two-dimensional fuzzy sets. This geometric interpretation of fuzzy sets guides to a redefinition of subsethood of fuzzy sets. Initially, a fuzzy set A was considered as a subset of fuzzy set B, if for all membership values $a_i \leq b_i$ is fulfilled. Kosko extended the concept to degrees of subsethood, hence fuzzifying subsethood itself. This degree is derived geometrically from the hyperrectangles assigned to the fuzzy sets. It equals the ratio of the volume of the intersection of both hyperrectangles to the volume of the container hyperrectangle. Thus, even the whole set is a subset of each of its "crisp" subsets to a certain degree[2].

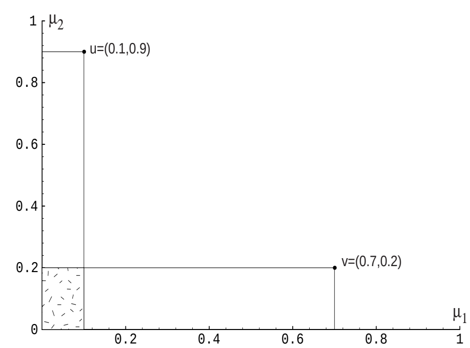

Figure 1: Fuzzy sets as points in the unit square.

Fuzzy subsethood can be used to define a new ranking scheme, which does not belong to one of the four categories of Barnett [1]. At first, we consider maxmin operations of the kind

$$\text{argmax}(\min(\vec{f}(p))$$

[2]This was considered as fuzzy foundation of probability by Kosko.

and

$$\operatorname{argmin}(\max(\vec{f}(p))).$$

If $\vec{f}$ is the identity operation, which preserves the vector p, both definitions, employed as a ranking scheme, would not lead to a new ranking scheme. While the minmax version would violate the dominance relation, the maxmin version merely is a reduced ordering. The parameter function here is the distance to the main diagonal of the unit cube, wherein the vector p resides.

However, when $\vec{f}$ is derived from fuzzy subsethood degrees of one p_i within the set of other p_j, the resultant ranking scheme is neither marginal nor reduced ordering. Moreover, it allows for the design of a generalized dilation. This will lead to the Fuzzy-Pareto-Morphology.

2.5 Fuzzy-Pareto-Morphology

Finally, we can give the definition of the FPM. If the neighborhood M of a pixel is given by n pixels with color values x_{ij} with $i = 1, \ldots, n$ and $j = 1, 2$ or $j = 1, 2, 3$ [3], then the set operation P is given by:

$$P \equiv \operatorname*{argmin}_i \left[\max_{k \neq i} \frac{\prod_j \min(x_{ij}, x_{kj})}{\prod_j x_{ij}} \right]. \tag{1}$$

The dilation of FPM is derived from this set function, which gives the replaced supremum value of a neighborhood M of a pixel. The accompanying erosion is given as the complement of this operation according to the set function:

$$P \equiv \operatorname*{argmax}_i \left[\min_{k \neq i} \frac{\prod_j \min(x_{ij}, x_{kj})}{\prod_j x_{ij}} \right]. \tag{2}$$

The FPM could be considered as a kind of competition among the color values mapped from a pixel by the mask M. The color values are considered as fuzzy sets[4]. Actually, they are fuzzy sets by the extension principle. The membership values indicates the degree, by which they are "white." Each color value enters into the competition by the maximum degree, by which it is a subset of other color values. Each color value, which is dominated by another color value, goes with the maximal possible value 1 into the competition. The winner of the competition is the one with the minimal maximum value. From this, it can be easily seen, that this definition comprises a Pareto-Morphology as well.

Some remarks about the definition:

- If one of the x_{ij} is 0, the fraction is not defined. In this case, the corresponding parts of nominator and denominator should be neglected, hence performing the competition "one dimension below." The question, whether it is useful to neglect the corresponding parts of other arguments of the maximum operation, wherein the division by zero occurred, will be left open in this paper.
- It is useful to restrict the derivation of the terms in equation 1 to different values of p, i.e. neglecting multiple occurrences of the same color value in M.
- The value in equation 1 depends on the chosen color model. This can not be prevented, for there will be no ranking independent of the color model. In some cases, several color values will give the same values (e.g. a neighborhood in an RGB-image with one point red, one point blue and one point green). From this, the argmin operation becomes ambiguous. This is no contradiction, because the approach does not prefer any one of the basic colors. In this case, a fixed arrangement of colors has to be taken as resultant color value, or a randomly chosen one.

[3] For the purpose of illustration, sometimes only two color-channels will be considered

[4] The color components should be scaled to the $[0, 1]$ interval in beforehand.

3 Properties of the FPM

From its definition, FPM is a Pareto-Morphology, because, as it was just remarked, all dominated points goes with value 1 into the competition for the minimal value of the maximum subsethood degree to all other points. The winner will be an element of the Pareto set.

The FPM is *not* a reduced ordering. This is the most important property, because it establishes FPM as a native color operation instead of a grayscale image processing based one. In order to proof this, consider figure 2. We assume, that there is a parameter function f, which gives the same ranking as the FPM ordering scheme given with equation 1. If we take the three points (1,10), (9,2) and (10,1), the selected point by FPM is (1,10). If we take the three points (1,10), (2,9) and (10,1), the selected element will be (10,1). But, both sets have two points in common, which exchange their ranking. If there is such an f, it must be $f(1, 10) > f(10, 1)$ from the first case, but also $f(1, 10) < f(10, 1)$ from the second case. This is not possible, hence there is not such a f.

For a better understanding of this fact, one has to consider, what is finally selected by the ordering scheme of FPM. It is the most "unusual" color value in the context of the other color values. The "strengths" of each color value is influenced by the presence of other color values. So, the alternating points (2,9) and (9,2) in the example just given, weaken the "strengths" of the color values, to which they are nearby, in the competition.

Nevertheless, for exactly two points it will be a reduced ordering. The parameter function for ordering is the product of the components of p, as follows from examining the expression of equation 1 for this case:

$$\operatorname*{argmin}_{\{1,2\}} \left(\frac{\prod_j \min(x_{1j}, x_{2j})}{\prod_j x_{1j}}, \frac{\prod_j \min(x_{2j}, x_{1j})}{\prod_j x_{2j}} \right).$$

However, both nominators are equal, hence the selection is equal to

$$\operatorname*{argmax}_{\{1,2\}} \left(\prod_j x_{1j}, \prod_j x_{2j} \right),$$

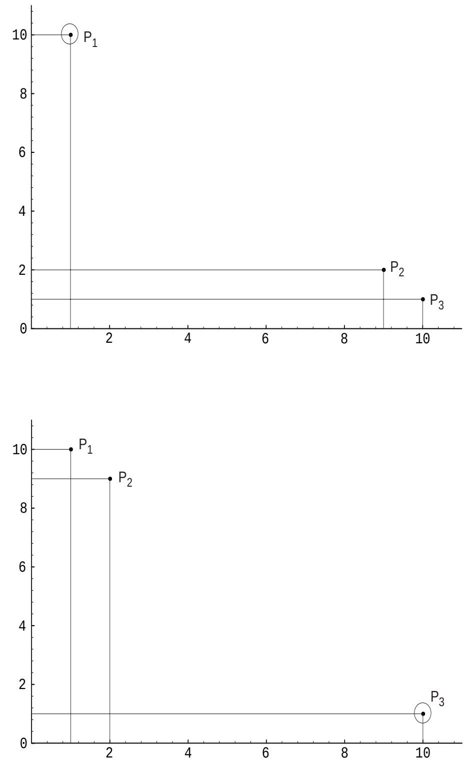

Figure 2: Counterexample for reduced ordering.

i.e. the product of components is the scalar functions needed for a reduced ordering.

FPM reduces to grayscale morphology, if each point is a scalar value. This follows directly from the Pareto property, because the Pareto set in the one-dimensional case has just one element, the maximum value. Because FPM selects from the Pareto set, it *must* take the maximum. This is the set function P of grayscale morphology.

Compatibility with binary morphology, i.e. requirement 3, is not generally fulfilled by FPM. Due to the following property of the Pareto set operator **P**

$$\mathbf{P}(A \cup B) = \mathbf{P}(\mathbf{P}(A) \cup \mathbf{P}(B)) \subseteq \mathbf{P}(A) \cup \mathbf{P}(B)$$

for two sets A and B, the Pareto set by itself is compatible. But, the ordering scheme could select different values.

To see, that FPM not necessarily selects the same value, consider the structuring elements given in figure 3. Because the left structuring elements contains two points, FPM is reduced ordering by the product of the components, as it was just mentioned. This means, the consecutive application of both structuring elements would select the color value with the largest product of its components out of the four positions covered by the structuring element on the right side of figure 3. But then, the FPM would be a reduced ordering according to the product of elements for four points, too! We just gave an example for three points for this to be impossible[5].

[5]For the fourth point, choose one which is dominated by one of our example, as (1,9) or so.

This demonstrates the fact, that FPM is not compatible with binary morphology.

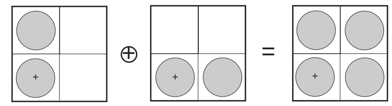

Figure 3: Counterexample for binary compatibility.

However, it is compatible in a practical sense. The chance of taking two different values out of the Pareto sets lowers with the number of elements of the Pareto set. The more "structure" is within the image, the smaller the Pareto sets will be. Violations of compatibility will become rare for cases, for which morphological image processing is intended. Random image parts, without any structure, will have large Pareto sets. But here, the actually taken color value by FPM ordering scheme will not be such important.

4 Other operations

Based on the concept of fuzzy subsethood, other operations for color image processing can be designed. Some examples will be given in the following:

- A color threshold operation can be defined, for which the result image is not a binary but a grayscale image. Given $p = (p_x, p_y, p_z)$, the grayvalue at each image position with color value v is the degree of subsethood of the color value v in p.
- A fuzzy color image subtraction operation of two images p_1 and p_2, by which the degree of fuzzy-subsethood of the color value of p_1 in the corresponding color value in p_2, multiplied by the original color value, is assigned to each image position.

  This is important for e.g. the definition of the morphological gradient in FPM. In grayscale morphology, the morphological gradient is the difference of dilated and eroded image by the same structuring element. However, simply subtracting two color values would introduce alien color values in the result image. By replacing subtraction with the mutual fuzzy subsethood operation, this could be prevented[6].

  With this operation, edge operators from grayscale morphology can be extended to color values.
- The selection scheme of equation 1 actually computes values, for which the argument leading to the minimal value is taken as result. However, the minimal value itself can be taken as a grayvalue, and a grayscale image can be constructed this way (the so-called M-image). These operation may support the processing of color images by the newly proposed FPM.

[6]Multiplying all components of a color value by the same scalar gives a physiologically similar appearing color.

Examples for using these operations are considered in the next section.

5 Examples

Our motivation to develop a suitable color morphology came from the processing of colorized textiles. Instead of a general procedure for processing either kind of a textiled surface, it was decided to provide a toolset of operations, by the combination of which complex problems could be solved. From grayscale texture analysis, mathematical morphology is well known to comprise such a toolset. Hence, this versatility should be preserved for a color morphology. In the following, some examples are given for the application of the FPM in order to solve visual inspection tasks.

Example 1 (see figure 4) demonstrates the detection of blots in a colorized texture. Figure 4a gives the original image part[7]. Figure 4b shows the result of FPM dilation with a structuring element of size 3×3, figure 4c the result of applying this operation twice, figure 4d for applying it three times in a sequence. Figure 4e gives the M-image of the third dilation, clearly indicating the blots.

Example 2 demonstrates the detection of a (synthetic) misaligned pattern on a textured background. Figure 5a is a part of a textile surface with a misalignment, figure 5b a similar region, but with correct alignment, used as reference. For both images, the morphological gradient by using FPM is computed (figures 5c and 5d), and the fuzzy subtraction is applied to the gradient images. The result is given in figure 5e, proving a clear indication for a misalignment.

In figure 6, the detection of thread faults in a color textile is demonstrated. The structural property of the thread's horizontal orientation is used by the FPM. The structuring element is a vertically oriented mask of size 7. If opening, i.e. dilation followed by erosion, is applied with this mask, the thread "vanishes."

Figure 7 gives two examples for the color threshold operator, as defined in the last section, and demonstrates the different treatment of different structures in the same image by using two different color values as such "thresholds."

6 Summary

In this paper, the Fuzzy-Pareto-Morphology (FPM) was proposed and its relation to general issues of color morphology was intensively discussed. Past theory of multivariate ranking defined four classes of multivariate ranking schemes, among them the class of reduced ordering (or total ordering) can be used for designing a color morphology, which is vector-preserving, i.e. which will not introduce new color values into the result image. Also, the concept of the Pareto set of multivariate data sets gives a means for replacing the supremum, which is used in grayscale morphology. A class of generalized morphologies, the so-called Pareto-Morphologies, was proposed, which includes generalized morphologies based on reduced ordering by a monotonic ordering function. A generalized morphology is a Pareto-Morphology, if its result does not change, when its computations on the pixel neighborhood are restricted to the Pareto set of this neighborhood of a pixel. This property extends the usually required property of a dilation to commute with the supremum.

The question, whether there is a Pareto-Morphology, which is not based on reduced ordering, got a positive answer with the proposal of the FPM. In order to design the FPM, the concept of fuzzy subsethood of fuzzy sets within other fuzzy sets was applied to color values.

The advantage of the FPM of being not based on reduced ordering is the more native color treatment of color images. It was shown, that FPM is vector preserving, too, and that it becomes the standard grayscale morphology, if it is considered for the case of one-dimensional data. However, the requirement of compatibility is not fulfilled for structureless (noisy) images.

Other operations can be designed, based on the FPM (e.g. opening, closing, morphological gradient), or based on the concept of fuzzy subsethood (fuzzy subtraction of two images, color value "thresholding"). Also, intermediate results of the computations of FPM can be re-used as new image processing operators (like the M-image).

The FPM's and its accompanying operation's versatility for solving complex color image processing tasks was demonstrated by some examples, which were taken from the field of color textiles fault detection.

Currently, we are studying the interplay of different color spaces with the FPM outcome, and the adaptation of the scalar function of reduced ordering based morphologies to the color appearance in a textile image.

[7] The arrow is just for marking the fault, but it is processed as well.

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